Technological Change, Firm Heterogeneity and Wage Inequality*

Guido Matias Cortes[†] Adrian Lerche[‡]

Uta Schönberg[§] Jeanne Tschopp[¶]

December 20, 2021

Abstract

We argue that skill-biased technological change not only affects wage gaps between skill groups, but also increases wage inequality within skill groups, across workers in different firms. Building on a heterogeneous firm framework with labor market frictions, we show that an industry-wide skill-biased technological change shock will increase between-firm wage inequality within the industry through four main channels: changes in the skill wage premium (as in traditional models of technological change); increased employment concentration in more productive firms; increased wage dispersion between firms for workers of the same skill type; and increased dispersion in the skill mix that firms employ, due to more sorting of skilled workers into more productive firms. Importantly, a simultaneous increase in the supply of skilled workers does not offset the technology-induced rise in inequality. Using rich administrative matched employer-employee data from Germany, we provide empirical evidence of establishment-level adjustments that are in line with the predictions of the model. We further document that industries with more technological adoption exhibit particularly pronounced adjustment patterns along the dimensions highlighted by the model.

*Cortes and Tschopp are grateful to the Social Sciences and Humanities Research Council of Canada for its financial support (Grant 430-2018-00686). Uta Schönberg is grateful to the European Research Council for its financial support (Grant Number 818992 (FirmIneq)). We thank the Research Data Centre of the German Federal Employment Agency at the Institute for Employment Research for providing data access. We are also grateful to numerous conference and seminar participants for valuable comments and suggestions.

†York University; gmcortes@yorku.ca.

‡Institute for Employment Research (IAB); adrian.lerche@iab.de.

§University College London, Centre for Research and Analysis on Migration (CReAM) and Institute for Employment Research (IAB); u.schoenberg@ucl.ac.uk.

¶University of Bern; jeanne.tschopp@vwi.unibe.ch.

1 Introduction

Income inequality has risen dramatically over the past decades in many high-income countries. The development of new technologies has been proposed as a key driver of this pattern. The literature has argued that technology has asymmetric impacts across different groups of workers, depending on their skill levels or the tasks that they perform (e.g. Katz & Murphy, 1992; Autor et al., 2003, 2006; Goos et al., 2014; Jaimovich & Siu, 2020). According to this view, technology has impacted wage inequality by changing the demand for different skills and tasks, thus changing the employment structure of the economy and the relative wage returns for different groups.

Intriguingly, however, a recent parallel literature has shown that most of the rise in wage inequality can be traced back to increasing wage differentials among observationally similar individuals, working in different firms (Card et al., 2013; Barth et al., 2016; Helpman et al., 2017; Song et al., 2019). This literature has argued that individual-level wages have become increasingly dependent on where people work, rather than the skills that they possess or the tasks that they perform.

While the literature on between-firm inequality has documented many novel empirical facts, it is not yet entirely clear what the driving forces behind these patterns are. In comparison, while the literature on technological change has provided deep insights into the evolution of relative wage gaps *across* skill and task groups, it has, by drawing on models of perfectly competitive labor markets with a representative firm, so far provided limited insights into the growing wage differentials observed *within* groups, across firms.

In this paper, we argue that the development of skill-biased automation technologies can account not only for increases in inequality between skill groups, but also for increases in inequality within groups, across workers in different workplaces, as observed in the data. We show this theoretically, using a rich yet tractable heterogeneous firm framework, and empirically, verifying the predictions of the model using administrative matched employer-employee data from Germany.

Our analysis draws on data from the Beschäftigtenhistorik (BEH) from the Institute for Employment Research (IAB), covering the universe of private sector workers and establishments in West Germany between 1990 and 2010. We supplement these data with information from the IAB Establishment Panel (IABEP), which provides measures of establishment-level sales and allows us to construct a measure of labor productivity for the establishments covered by the survey.

In line with existing evidence, we document a substantial increase in the share of skilled workers in total employment. In contrast, the skilled wage premium has remained roughly constant in Germany. The increase in the supply of skilled labor thus appears to have roughly offset the demand effects on the skilled wage premium coming from skill-biased technological change. In consequence, essentially all of the increase in wage inequality observed in Germany between 1990 and 2010 is due to growing wage differentials among workers *within* the same broad skill group.

In line with the literature, we also find that the rise in overall wage inequality is primarily due to growing wage differences between, rather than within establishments. We further show that, in any given year, about 60% of the overall variance of between-establishment log-wages is accounted for by pay differences across establishments within 3-digit industries. This finding indicates that we are far from having a 'representative establishment', in terms of pay and skill structure, even within detailed industries. Moreover, this heterogeneity between establishments within industries has been growing over time and accounts for the majority of the overall rise in between-establishment wage inequality. In this paper, we focus on this growing heterogeneity in (log) wages across establishments within detailed industries.

In order to explore the potentially heterogeneous impacts of skill-biased technological change across firms within industries, we consider a version of the model in Helpman et al. (2010), in which firms within industries differ in terms of their productivity as well as their technology of production (i.e. their optimal mix of skilled and unskilled workers). In equilibrium, more productive firms find it optimal to employ more workers of both types, have a higher skilled employment share, and pay higher wages (overall and conditional on skill type).

We then introduce an aggregate skill-biased technological change (SBTC) shock in the spirit of Katz & Murphy (1992) and Autor et al. (1998) within this rich heterogeneous firm setting, and analyze its impact across workers and firms. We focus on the relative (rather than absolute) effects of the shock (e.g., the wage impacts for workers in more productive relative to less productive firms). In spite of being an aggregate shock that is common across all firms, SBTC leads to an increase in between-firm wage inequality. This occurs through a number of key channels. The first is an increase in the skilled wage premium (the wage of skilled relative to unskilled workers). This is similar to the channel that arises in traditional models of SBTC which feature a competitive labor market and a homogeneous representative firm. In our setting with heterogeneous firms, the change in the skilled wage premium not only increases overall wage inequality (due to a larger gap in wages *between* skill groups), but also wage inequality between firms within industries, as firms with different productivity levels employ different shares of skilled and unskilled workers.

The remaining channels through which SBTC impacts inequality are novel to our setting. First, the model predicts that SBTC leads to differential employment growth, whereby the more productive, higher-paying firms in the industry are predicted to grow more. This leads to a rise in employment concentration, which contributes to an increase in worker-weighted measures of between-firm wage inequality. Second, the model predicts that SBTC leads to an endogenous increase in worker segregation by skill, driven by increased sorting of skilled workers to high-productivity (and hence high-wage) firms. Thirdly, the model generates endogenous within-firm wage changes, with more productive firms disproportionately increasing the wage that they pay to workers of each skill group, thus further contributing to the increase in between-firm wage inequality.

Our framework additionally implies that SBTC will increase the productivity threshold for production, which results in the exit of low-productivity firms. This change in the composition of firms increases the variance of productivity among operating firms, and may further increase the variance of (log) firm wages.

Traditional models of SBTC posit that there is an ongoing "race" between technological change – which raises the demand for skilled workers and leads to a rise in the skill premium and wage inequality, all else equal – and increases in educational attainment – which raise the supply of skilled workers and therefore dampen the effects of technological change (Tinbergen, 1974, 1975; Katz & Murphy, 1992; Goldin & Katz, 2008; Acemoglu & Autor, 2011). In our framework, an increase in the supply of skilled workers may dampen the effect of SBTC on the skill premium, just as in the traditional models. However, the impact of the increased supply on the other channels highlighted by our model is more nuanced. In fact, we show that the increased supply will in general *amplify* (or at least not fully offset) the effects of SBTC on between-firm wage inequality. Hence, even if the increase in the supply of skilled workers offsets the impact of SBTC on the skill premium, between-firm wage inequality may still grow due to differential changes in employment and wages across different types of firms.

Guided by the model, we return to the BEH and IABEP data and verify its key predictions. Consistent with the presence of ongoing SBTC (potentially compounded by the rising supply of skilled workers), we find that the within-industry establishment-level associations between productivity, employment, skill shares and wages have become stronger over our sample period. For example, while a 1% increase in the establishment's labor productivity was associated with a 0.1% increase in establishment size (i.e., number of employees) in the early 1990s, this association quadrupled to 0.4% in 2010. Similarly, while a 1% increase in establishment size was associated with an increase of 0.06% in the wage establishments pay to workers of the same skill type in the early 1990s, this association increased to 0.09% in 2010.

In line with this evidence, when considering longitudinal changes within establishments, we find that larger establishments tend to pull further away from smaller establishments in the same industry, by experiencing faster productivity growth, employing more skilled workers, and increasing the wages that they pay to workers of a given skill group. Moreover, in line with recent evidence on increased employment concentration (e.g. Autor et al., 2020), establishments that are more productive, employ more skilled workers or pay higher wages at baseline experience higher employment growth than other establishments in the same industry, a pattern that is consistent with the implications of an aggregate SBTC shock in the model (potentially compounded by the rising supply of skilled workers).

We further show that, in line with the model, establishments within industries have become increasingly heterogeneous in terms of their skill mix (rather than converging to more similar technologies of production over time). This is driven by increased sorting of skilled workers towards establishments that pay high wage premiums conditional on worker skill. In consequence, workplace segregation has increased such that workers of the same skill group are increasingly clustered in the same workplaces. This pattern is in line with the literature that has demonstrated that segregation of high-wage workers in high-wage firms is an important proximate reason behind the rise in between-firm wage inequality (Card et al., 2013; Song et al., 2019). Here, we can rationalize this pattern as being driven by an aggregate technological change shock, potentially compounded by the rising supply of skilled workers.

We then perform a series of decompositions that allow us to assess the relative importance of the different channels that drive the increase in between-establishment wage inequality according to the model. First, we determine the role of entry and exit relative to changes among continuing establishments. We find that changes in establishment composition due to entry and exit have led to a small increase in between-establishment wage inequality; changes among continuing establishments, however, are quantitatively much more important.

Second, we analyze the role of sorting along skill dimensions. When considering only two skill groups, we find that changes in the sorting of skilled workers towards high wage premium establishments account for 12% of the change in the wage variance among continuing establishments within industries. Workplace segregation, while present, plays only a minor role in the increase in between-establishment inequality, accounting for an additional 3% of the change.[1] The remainder of the change is due to an increase in the variance of the wage premiums that establishments pay conditional on worker skill. Around half of the rise in this variance is due to differential employment growth across establishments. Thus, an important reason for the rise in between-establishment wage inequality is the fact that

[1]We also perform the decomposition based on detailed occupational categories, and find that, at that level of detail, sorting and segregation account for about 25% and 20% of the increase in within-industry between-establishment wage inequality among continuing establishments.

establishments that paid higher wages at baseline have expanded in size relative to lower paying establishments – a channel that the literature on wage inequality has so far ignored.

As a final exercise, we provide direct evidence of the link between technological change and the establishment-level patterns that we have identified. We do this by leveraging variation across industries in technology adoption, which we measure in three different ways: based on the change in each industry's skilled employment share over our sample period; based on industry-level robot adoption data from the International Federation of Robotics; and based on ICT capital usage data from EUKLEMS. Our key finding is that industries that have adopted more technology have experienced disproportionate increases in between-establishment wage inequality (both overall and within skill groups), skilled share heterogeneity, and the sorting of skilled workers to high-wage establishments. This pattern persists when we control for differential trade and offshorability exposure across industries, thus corroborating the importance of technology adoption (potentially exacerbated by increases in the supply of skilled workers) in driving the establishment-level patterns that we have documented.

Our findings make important contributions and connect to several strands of the literature. First, we provide an important innovation to the literature that studies the impacts of technological change on wage inequality. This literature has primarily relied on representative firm frameworks with perfect competition, and has thus solely focused on the impacts of technological change on inequality that operate via changes in the skill or task structure of the economy (see e.g. Katz & Murphy, 1992; Machin & Van Reenen, 1998; Acemoglu & Autor, 2011; Michaels et al., 2014; Autor et al., 2015; Akerman et al., 2015; Graetz & Michaels, 2018; Dauth et al., 2021).

By embedding an SBTC shock within a framework composed of heterogeneous firms with imperfect competition, we are able to show that skill-biased technological change not only differentially affects wages of workers with different skill levels, but can also account for the quantitatively much more important rise in inequality within skill groups, for workers in different firms. As our framework and empirical analysis demonstrate, an industry-wide technology shock has very different impacts on different firms in the industry, with some firms exiting the market and other firms expanding and increasing their wages. Our results thus paint a much richer picture about the individual- and firm-level impacts of skill-biased technological change, by highlighting that the relative impact across individuals will depend not only on their skill level, but also on the type of firm that they are matched to. Moreover, a key implication of our framework is that there is no "race" between technology and education – an expansion in the supply of skilled workers may in fact exacerbate rather than dampen the effects of technological change on between-firm wage inequality.

Our analysis also provides an important contribution to the literature on the rise in between-firm wage inequality (Card et al., 2013; Song et al., 2019; Barth et al., 2016; Helpman et al., 2017; Criscuolo et al., 2020). This literature has been very successful in highlighting the increasing importance of firms for individual wages, and has documented increases in worker sorting and segregation (e.g. Kramarz et al., 1996; Cortes & Salvatori, 2019; Wilmers & Aeppli, 2021). It has, however, been more limited in terms of characterizing the underlying driving forces behind these patterns. We provide a tractable theoretical framework that allows us to study the interplay between skill-biased technological change and these important workplace-level patterns at the industry level. Guided by the model, we document new empirical findings regarding the strengthening associations between various workplace-level outcomes, as well as quantifying the role of establishment entry and exit, sorting and segregation along skill dimensions, and differential employment growth across establishments for wage inequality.

Our paper further contributes to the literature on the rise in concentration and the increased dominance of so-called superstar firms (Autor et al., 2017, 2020; Azar et al., 2020a,b; Bajgar et al., 2019). We show that skill-biased technological change leads to the disproportionate employment growth of the most productive workplaces within an industry. Technological change may therefore be at least partly responsible for the rise in employment concentration. We also show how the employment shift towards 'superstar' establishments directly contributes to the rise in wage inequality: even in the absence of any wage changes within establishments, the fact that more productive, higher wage workplaces have experienced differential employment growth has led to an increase in worker-weighted measures of between-establishment wage inequality.[2]

Our paper is also related to a number of recent studies which investigate the impact of *firm-level* adoption of industrial robots (e.g. Acemoglu et al., 2020; Bonfiglioli et al., 2020; Koch et al., 2021), automation expenditures (Bessen et al., 2020; Aghion et al., 2020) or innovation (Lindner et al., 2021) on firm-level outcomes such as employment, sales, value added per worker, total factor productivity or skill intensity. These studies generally find that technology adoption is associated with increases in employment, sales and skill intensities at the firm level. While we are also interested in the firm-specific impacts of technological advances, out study highlights that an *industry-wide* shock can have differential effects between firms.

The paper most related to ours is Haanwinckel (2020).[3] While we share a similar goal

[2]See Webber (2015); Mueller et al. (2017); Rinz (2020) and Cortes & Tschopp (2020) for more detailed analyses of the link between rising concentration and rising wage inequality.

[3]Other papers in the literature have provided a rich analysis of how technology affects the sorting of workers to jobs (e.g. Lindenlaub, 2017). However, these types of models often have no natural definition of a firm and assume that worker types are perfect substitutes in production.

in terms of understanding the role of technological change and changes in the supply of skills for wage inequality, our framework and our approach are substantially different. In the Haanwinckel (2020) model, workers have idiosyncratic tastes for different workplaces, thus giving employers wage-setting power (as in e.g. Bhaskar et al., 2002; Card et al., 2018). In contrast, our model generates wage heterogeneity due to search and matching frictions and match-specific worker ability. Hence, firms that pay higher wages in our model do so for reasons that are directly related to productivity. Our model also yields closed-form solutions for the key equilibrium outcomes of interest, which are supported by our motivating empirical evidence. These solutions allow us to derive clear comparative statics results that illuminate the intuition behind the mechanisms through which changes in technology and changes in the supply of skilled workers lead to changes in inequality in the model. Haanwinckel (2020) uses the setup of his model in order to perform a quantitative exercise that disentangles the role of different shocks (including skill-biased technological change, changes in the supply of skilled workers and minimum wages) for overall wage inequality in Brazil. We instead highlight the importance of focusing on the rising heterogeneity in wages between workplaces within industries in Germany, and we provide novel empirical evidence regarding the micro-level adjustments underlying these changes, which we can directly link to our model mechanisms. We also exploit variation in technology adoption across industries in order to provide further evidence of the role of technological change for between-workplace inequality.

2 Data

2.1 Social Security Records (Beschäftigtenhistorik (BEH))

Our main data are drawn from social security records provided by the Institute for Employment Research in Nuremberg (IAB) – the so-called Beschäftigtenhistorik (BEH, 2016 version).

We focus on developments after 1990 when wage inequality started to increase sharply in Germany across the entire distribution of wages (see for example Dustmann et al., 2014; Card et al., 2013). Due to structural breaks after 2010 in key variables such as occupations and workers' full-time status, we end the analysis in 2010. The data source comprises all men and women covered by the social security system – roughly 80% of the German workforce. Not included are civil servants, the self-employed, and military personnel.

Our data source offers some key advantages. A first advantage is its large size, allowing us to accurately capture trends in wage inequality even within detailed industries. Second, our data contain comprehensive and accurate information on a number of worker and establishment characteristics that are not always included in other administrative data sources, such as workers' occupation, employment status and wages (which always refer to a single establishment and are never averaged across establishments) and establishments' industry affiliation. Importantly, unique establishment identifiers allow us not only to decompose overall wage inequality into a within and a between establishment component, but also to study (changes in) establishment heterogeneity within industries more broadly. Establishment identifiers further allow us to paint an accurate picture of entry and exit across industries and time.

From this data source, we select all full- and part-time employment spells that refer to June 30 of each year. We restrict the sample to workers who are currently not in an apprenticeship, are aged between 16 and 65, and are employed in West Germany. We exclude industries in the primary sector and some small industries such as private households and international organizations. We further drop workers with missing occupation, missing employment status, or implausibly low wages below the limit for which social security contributions have to be paid, as well as establishments with missing industry affiliation and establishments employing only part-time workers. These sample restrictions affect less then 1% of all worker-level observations.

As is common in administrative data sources, wages are censored at the highest social security limit, affecting on average about 8% of observations. We follow Dustmann et al. (2009) and Card et al. (2013) and impute censored wages, assuming that (log) wages are normally distributed with heterogeneous variances that vary by year, age, education and sex; see Appendix A.1 for details. We deflate wages using 1995 as the base year.

In order to classify individuals as either skilled or unskilled, we make use of information on their occupational affiliation, rather than their education level. This is due to the fact that, in Germany, nearly 70% of individuals in a school leaving cohort undergo apprenticeship and vocational training; in consequence, the share of university graduates is smaller than in many other developed countries, and occupations that are generally regarded as being highly skilled tend to be performed by individuals with apprenticeship training as well as university graduates. We therefore classify an individual as being highly skilled if they are working in a professional, managerial or technical occupation.[4] We label workers in all other occupations (i.e. those in administrative and clerical jobs, production jobs, and personal service occupations) as unskilled.[5] Appendix Table A.1 provides details on the mapping of occupation codes to skill groups. The two most common skilled occupations are nurses and

[4]These groups correspond to what has been generally referred to as "abstract" occupations in the literature on labor market polarization (e.g. Autor et al., 2003; Acemoglu & Autor, 2011).

[5]Below we also present results that distinguish between the 317 detailed occupational categories that are available in the data, rather than distinguishing only between these two broad groups.

managers, accounting for more than 15% of all skilled workers. The most common unskilled occupation is office clerks, comprising 16.9% of all unskilled workers. The table also shows the close mapping to education levels. Whereas 37.0% of all workers in skilled occupations hold a university degree, this is the case for only 6.8% of workers in unskilled occupations.

For the main empirical analysis, we aggregate the worker level information to the level of the establishment (by year). We thereby create an establishment panel which records industry affiliation, and tracks entry and exit, size, employment share of skilled workers, and average wages in each establishment over time (overall and by skill group). Our employment counts include part-time workers with a weight of 0.5. Since we do not observe hours worked, our measures of establishment wages are based on full-time workers only.[6]

Our industry classification refers to 3-digit NAICS codes which distinguish between 196 industries. Due to a structural break in the industry classification in the social security data in 1999, we harmonize the industry classification as described in Appendix A.2.

2.2 The IAB Establishment Panel (IABEP)

Since the social security records drawn from the BEH do not contain information on establishment outcomes such as total sales or labor productivity, we augment the social security records with data from the IAB Establishment Survey (IABEP). The IABEP survey was first administered in 1993 to 4,265 West German establishments. By 2010, the number of surveyed establishments had increased to over 16,000. From this database, we select all West German establishments with at least one full-time employee that participated in the IABEP at least once. Adopting the same sample selection criteria as in the social security records (BEH), we drop establishments with missing industry affiliation as well as establishments in the primary sector and some smaller sectors such as private households and international organizations. Using the unique establishment identifiers, we then merge information from the BEH social security records to the IABEP. We compute an establishment's labor productivity as total sales (obtained from the IABEP), divided by the number of full-time equivalent workers (obtained from the BEH). In the empirical analysis based on the IABEP, we use the weights provided by the survey in order to guarantee representativeness for workers.

[6] Fitzenberger & Seidlitz (2020) provide evidence that a fraction of part-time workers are misclassified as full-time workers. Even though this affects inequality measures in a given year, the authors show that this misclassification is not driving the rise in inequality over time.

2.3 Industry-Level Technology Adoption Measures

We supplement these two main data sources with industry-level data on technology adoption. First, following Graetz & Michaels (2018) and Acemoglu & Restrepo (2020), we use data on robot usage from the International Federation of Robotics (IFR).[7]

Second, we use data on the adoption of capital related to information and communication technologies (ICT) from the EUKLEMS data set. We use data from the November 2009 release, which uses ISIC revision 3 industry codes which can be matched to the industry codes in the BEH social security data at the 2-digit level. Our measure of ICT assets is based on the real fixed capital stock of computing and communication equipment, and computer software.

3 Motivating Evidence

This section presents some empirical facts related to the evolution of the skill structure of employment and wage inequality in Germany between 1990 and 2010. These empirical patterns motivate the setup of our theoretical framework and the subsequent analysis of the impact of technology on (between-establishment) wage inequality.

Skill Composition of Employment, Skill Premium and the Importance of Inequality within Skill Groups. A large literature has documented the rising employment share of skilled workers and skilled occupations across many developed countries (see e.g. Katz & Murphy, 1992; Autor et al., 1998; Acemoglu & Autor, 2011). Panel A of Figure 1 verifies this pattern for Germany, and shows that the aggregate employment share of skilled workers steadily rose from about 20% in 1990 to more than 26% in 2010 – a rise of 34% over two decades. This increase was in part driven by differential industry growth: industries which employ a larger share of skilled workers grew at a faster rate than industries which predominantly employ unskilled workers. Yet, even when keeping the industry structure constant at 1990 levels (the grey dashed line; see Appendix B.1 for details), the employment share of skilled workers rose substantially by about 18%.

Whereas the skilled employment share strongly increased between 1990 and 2010, Panel B of Figure 1 shows that the skill premium (i.e., the wage gap between skilled and unskilled

[7] A robot is defined as an "automatically controlled, re-programmable, and multipurpose machine" and as "fully autonomous machines that do not need a human operator and that can be programmed to perform several manual tasks such as welding, painting, assembling, handling materials, or packaging." We use the crosswalk provided by Dauth et al. (2021) to match industry codes in the robot data to industry codes in the BEH social security data.

workers; see Appendix B.2 for details) remained roughly constant over our sample period.[8] This suggests that the rise in demand for skilled workers driven by skill-biased technological change was, in Germany at least, largely offset by an expansion in the supply of skilled workers (due e.g. to increased educational attainment or increased apprenticeship training). It also implies that there will be limited scope for changes in wage inequality *between* skill groups to account for much of the rise in overall inequality.

We confirm this in Panel C of Figure 1. The black circles show that overall wage inequality in West Germany, measured as the variance of individual-level log wages, rose sharply from the mid-1990s onwards, from 0.195 in 1995 to 0.276 in 2010, a 41.5% increase (see also Dustmann et al., 2009, 2014). The light grey diamonds, meanwhile, indicate that the increase is almost entirely a within-group phenomenon (see Appendix B.3 for details of the computation of wage inequality within skill groups). In other words, essentially all of the increase in wage inequality is driven by increased wage heterogeneity among workers in the same skill group. Although the literature on skill-biased technological change (SBTC) focuses on changes in inequality that operate through the between-skill component (as this is the only relevant dimension in a representative firm framework with a perfectly competitive labor market), this component – which is equal to the gap between the black circles and the light grey diamonds in the Figure – remained stable in Germany over this time period.

The grey triangles in Panel C of Figure 1 reflect the evolution of the within-*occupation* variance of log-wages (using 317 occupational categories). These detailed occupational categories provide us with a much finer proxy for the skill requirements of workers' jobs. The results support the conclusions drawn from the analysis with two broad skill groups. Although the within-occupation variance is much lower than the within-skill variance, wage differences within detailed occupational categories account for the majority of the overall log-wage variance in the cross-section, and account for more than half of the increase in the variance over time. Thus, there is substantial heterogeneity in wages across workers within the same detailed occupations, and wage inequality rose sharply also within these detailed occupational groups.

Wage Inequality Within and Between Establishments. The finding that essentially all of the increase in inequality is due to increasing wage differences among workers in the same skill group does not mean that SBTC is not an important driver of wage inequality in Germany. As we show below, in a setting that departs from the traditional representative firm framework with perfect competition in the labor market, SBTC can lead to an increase

[8] The solid black line shows the evolution of the skill premium when industry-specific skill premiums are averaged across industries using the contemporaneous industry structure. The dashed grey line plots the average of the industry-specific skill premiums using the 1990 industry structure.

in inequality across workers in different workplaces (even if the supply of skilled workers keeps up with the rise in demand). Panel A of Figure 2 shows that the increase in overall wage inequality is nearly entirely driven by increasing wage differences between establishments – a pattern that is in line with the broader evidence in the literature regarding the increasing importance of between-firm wage differentials, documented by e.g. Card et al. (2013); Song et al. (2019); Barth et al. (2016); Helpman et al. (2017). The figure decomposes the variance of individual log wages, denoted Var_t, into a within-establishment and a between-establishment component as follows:

$$\begin{aligned} Var_t &= \frac{1}{n_t}\sum_i(\ln w_{it} - \overline{\ln w_t})^2 \\ &= \underbrace{\frac{1}{n_t}\sum_f\sum_{i\in i_{ft}}(\ln w_{it} - \overline{\ln w}_{ft})^2}_{\text{within establishments}} + \underbrace{\frac{1}{n_t}\sum_f n_{ft}(\overline{\ln w}_{ft} - \overline{\ln w}_t)^2}_{\text{between establishments } (Var_t^{BE})}, \end{aligned} \tag{1}$$

where i denotes an individual and f indexes establishments. $\ln w_{it}$ is the log wage of individual i at time t, $\overline{\ln w}_t$ is the average log wage in period t, and $\overline{\ln w}_{ft}$ is the average log wage in establishment f in period t. n_t is the total number of workers and n_{ft} is the total number of workers at establishment f in year t (i_{ft} denotes this set of individuals). The figure highlights that the increase in the log-wage variance observed between 1990 and 2010 occurred almost entirely between establishments; increases in within-establishment wage differentials account for only 1.6% of the rise.

Panel B of Figure 2 explores the extent to which between-establishment wage differentials, as captured by Var_t^{BE} in Equation (1), are due to differences between establishments in the same 3-digit industry or due to differences between establishments in different industries:

$$Var_t^{BE} = \underbrace{\frac{1}{n_t}\sum_k\sum_{f\in f_{kt}} n_{ft}(\overline{\ln w}_{ft} - \overline{\ln w}_{kt})^2}_{\text{between establishments, within industries}} + \underbrace{\frac{1}{n_t}\sum_k n_{kt}(\overline{\ln w}_{kt} - \overline{\ln w}_t)^2}_{\text{between industries}}, \tag{2}$$

where k indexes industries, f_{kt} is the set of establishments in industry k in year t, n_{kt} is the total number of workers in industry k and year t, and $\overline{\ln w}_{kt}$ is the average log wage in industry k at time t.

While both of the components are important, within-industry differences account for more than half of the between establishment variance in the cross-section, and more than half of its change over time (compare the black dots and the light-grey diamonds in Panel B). These results indicate that there is heterogeneity in pay across establishments that goes well beyond the differentials predicted by their industry affiliation, implying that we are far from having a 'representative establishment', in terms of pay, even within detailed industries. Moreover, this heterogeneity between establishments within industries has been growing over time and is an important driver of the overall rise in wage inequality. It is these within-industry differences across establishments that are the focus of our paper.[9]

The within-industry increase in between-establishment wage inequality could in principle be driven by industries with higher between-establishment wage inequality growing at a faster rate than the average industry. To rule out this possibility, the mid-grey triangles in Panel B of Figure 2 display the counterfactual within-industry increase in the variance of (log) establishment wages holding the industry structure constant at its 1990 level; see Appendix B.4 for details. While the counterfactual increase in between-establishment within-industry wage inequality is slightly less pronounced than the actual increase – indicating that industries with above average within-industry between-establishment wage variances have grown in relative terms – there is a clear increase also in counterfactual inequality between 1990 and 2010 of about 40%.

In order to link the findings on the importance of wage differentials between establishments to the findings on the importance of wage differentials *within* skill and occupation groups (as shown in Panel C of Figure 1), we compute a set of *establishment wage premiums*. These premiums capture differences in wages across establishments for workers within the same skill group or occupation, and are computed as follows (see Appendix B.5 for further details): First, we estimate, separately for each year, a regression of individual-level log wages on a skill indicator interacted with a full set of 3-digit industry fixed effects (thereby allowing for different skill wage premiums across industries and years). We then compute the average residual for each establishment in each year. This provides us with a time-varying measure of the premium paid by each establishment in each industry, given the skill composition of its workforce.

Since workers may substantially differ within broad skill groups, we also compute a second establishment wage premium by estimating, separately for each year, a regression of individual-level log wages on a full set of 3-digit occupation fixed effects (317 occupations) interacted with a full set of 3-digit industry fixed effects, and then calculating the average residual for each establishment in each year. This second establishment wage premium shows whether different establishments within a given industry pay different wages to workers within the same detailed occupation group, allowing us to rule out the possibility that the

[9] Recent work by Haltiwanger et al. (2021), in contrast, focuses on the importance of the between-industry component in accounting for the rise in between-firm wage inequality in the U.S. Their study finds that in the U.S., 25% of the increase in between-firm wage inequality over the past three decades occurred within very detailed 4-digit industries.

establishment wage premium computed based on the two broad skill groups solely reflects differences in the occupational structure within skill groups across establishments.

An alternative measure of the establishment wage premium would be the establishment fixed effect from an AKM-style wage regression that additionally conditions on worker fixed effects. While this measure allows for time-invariant worker heterogeneity even within detailed occupations in the establishment, it assumes that establishment premiums are constant over time (over the estimation window). Our goal, however, is to investigate whether and how establishment premiums have changed over time, and hence an AKM-style establishment premium would not be suitable for our purposes.

Panel C of Figure 2 plots the evolution of the within-industry between-establishment variance of log wages (based on the 1990 industry structure), along with the evolution of the variance of our estimated establishment wage premiums, based on the two broad skill groups and based on detailed occupations. The figure clearly highlights that heterogeneity in establishment wage premiums are a major component of within-industry between-establishment inequality, and these premiums have become increasingly dispersed over time, regardless of whether they are computed conditional on workers' broad skill levels (the light-grey diamonds in the figure) or on their detailed occupations (the mid-grey triangles).

Finally, in order to verify that rising heterogeneity between establishments is an important driver of the rise in wage inequality among both skilled and unskilled workers, Panel D of Figure 2 displays the evolution of the within-industry between-establishment variance of log wages (based on the 1990 industry structure) separately for each of the two groups. While establishment wages of skilled workers are generally more dispersed than those of unskilled workers, the increase in dispersion over time is similar for the two skill groups.

Importance of Establishment Premiums Relative to Skill or Occupational Premiums. To further gauge the relative importance of establishments versus skills or occupations in determining individual workers' wages, Panel A of Figure 3 displays the difference between the 90th and 10th and the 80th and 20th percentile in within-industry establishment wage premiums (based on the two skill groups) over time alongside the skilled wage premium, averaged across industries using 1990 industry employment shares. Whereas the 80th-20th and 90th-10th gaps in establishment wage premiums amounted to about 30 and 49 log points in 1990, they built up to 39 and 62 log points in 2010, an increase of 9 and 13 log points, respectively. For comparison, the (within-industry) skilled wage premium fluctuates around 40 log points over the same period. Hence, the wage differentials between high- and low-premium establishments for workers of a given skill level are large and are becoming even larger over time, whereas the gap in average wages between skilled and unskilled workers has remained relatively stable.

Panel B of Figure 3 reveals a similar picture when we focus on establishment wage premiums that account for each establishment's occupation structure at a detailed level (and not only their broad skill structure), and contrasts these to the observed wage differences between detailed occupational groups. Whereas the wage gap between the 10% lowest and 10% highest paying occupations increased by about 5 log points between 1990 and 2010, the wage gap between the 10% lowest and 10% highest paying establishments within an industry for workers in the same occupation increased by more than 10 log points over the same period. The increase is similarly larger across establishments than across occupations if we focus on the 80-20 gaps.

Overall, Figure 3 clearly illustrates that wages have become increasingly dependent on where workers work and (in relative terms) less dependent on workers' skills or the tasks that they perform.

Establishment Productivity, Skill Shares and Wages. As a final piece of motivating evidence before setting up our theoretical framework, we explore the cross-sectional link between productivity, size, skill shares and wages at the establishment level. Using sales data from the IABEP, Panel A of Table 1 explores the relationship between establishments' log productivity (total sales per full-time equivalent worker) and various establishment-level outcomes. This is analyzed by running a set of regressions which include fully interacted 3-digit industry and year fixed effects, so that identification is limited to cross-sectional variation within industry-year cells. Observations are weighted by establishment size and survey weights, to make results representative for workers. Standard errors are clustered at the establishment level.

Columns (1) and (2) of Panel A show that more productive establishments employ more workers – both skilled and unskilled – and hence are larger in terms of total employment.[10] The coefficient for skilled employment in Column (1) is larger than the one for unskilled employment in Column (2), suggesting that more productive establishments have a higher skilled employment share – a pattern that we verify directly in Column (3).

Column (4) shows that more productive establishments pay, on average, higher wages. In Column (5), we regress the establishment's wage premium (based on the two skill groups) on (log) establishment productivity. While the coefficient is positive – indicating that more productive establishments pay higher wages conditional on worker skill – it is smaller in magnitude than in Column (4), which is in line with more productive establishments employing

[10] For the analysis in these two columns, establishments with no workers of a given type are imputed to have one part-time worker (i.e., 0.5 full-time equivalent workers) of that type in order to be able to compute log employment.

more skilled workers. Column (6) confirms that more productive establishments pay higher wages also to workers within the same detailed occupation group; hence, the reason why more productive establishments pay higher wages is partly, but not entirely due to the fact that they employ a higher share of workers in higher paying occupations.

Finally, in Columns (7) and (8), we analyze the association between establishment productivity and wages separately for skilled and unskilled workers. To this end, we regress the average log wage of skilled and unskilled workers in the establishment on log establishment productivity (plus the full set of industry-year fixed effects), restricting the sample to establishments that employ workers of both types. While the point estimates indicate that an increase in productivity is associated with a slightly larger increase in unskilled than skilled wages, the difference between the coefficients for the two types of workers is not statistically significant at conventional levels. We thus conclude that in higher productivity establishments, wages tend to be higher for both skilled and unskilled workers in similar proportions.

Panel B of Table 1 uses log establishment size as the key regressor of interest. Column (1) confirms the positive and statistically significant relationship between establishment size and establishment productivity – in line with the evidence in Columns (1) and (2) of Panel A. The remaining columns of Panel B draw on the full BEH records. In Column (2) we show that larger establishments employ a higher share of skilled workers. Larger establishments also pay higher wages on average not only overall (Column (3)), but also to workers of the same skill group (Column (4)), and to workers within the same detailed occupation (Column (5)). The results in Columns (6) and (7) further highlight that, within industries, larger establishments pay higher wages to both unskilled and skilled workers in similar proportions: An increase in establishment size of 1% is associated with an increase of 0.061% for skilled workers, and 0.060% for unskilled workers.[11]

To summarize, we observe an empirical link between establishment productivity, size, skill composition and wages. These patterns motivate our model setup. Note that these relationships occur within 3-digit industries and are thus not accounted for by differences across industries in establishment sizes, wages or productivities.

4 Theoretical Framework

In this section we set up a theoretical framework that helps guide our analysis of the link between skill-biased technological change and between-firm wage inequality. Motivated by

[11] While the difference between the two coefficients is statistically significant at a 5% level, it is very small in magnitude.

the evidence in the previous section, we set up a model that allows for wage heterogeneity between firms within industries, with a production structure that distinguishes between two skill groups, and where firm productivity and skill usage are linked. We use the model in order to investigate the ways in which skill-biased technological change may impact wage inequality between firms within industries.

We consider a version of the framework developed by Helpman et al. (2010) – a rich, yet tractable model of firm heterogeneity that allows for wage differentials across firms for the same worker type within industries. Helpman et al. (2010) extend the Melitz (2003) model by deviating from the benchmark of perfect competition in the labor market, instead introducing Diamond–Mortensen–Pissarides search and matching frictions (Diamond, 1982a,b; Mortensen & Pissarides, 1994), as well as match-specific ability heterogeneity and a screening technology. We focus on the closed economy version of the extension of the model that allows for two types of labor inputs (Section 5.2 of their paper) which, in our setting, we think of as two different skill groups (skilled and unskilled).

In Sections 4.1 and 4.2, we briefly outline the key components of the model and the equilibrium conditions, as derived by Helpman et al. (2010).[12] Our key innovations relative to the analysis in Helpman et al. (2010) are in Sections 4.3 and 4.4. In Section 4.3 we consider the implications of an aggregate skill-biased technological change shock in the spirit of Katz & Murphy (1992) and Autor et al. (1998). As in that literature, we model SBTC as an exogenous aggregate change in the factor-augmenting parameter associated with skilled workers, and study the implications for various workplace-level and industry-level outcomes. Given the evidence that the supply of skilled workers has also increased over time (see our discussion above), in Section 4.4 we consider whether the implications of skill-biased technological change are mitigated or amplified by a simultaneous expansion in the supply of workers of this type.

4.1 Overview of the Helpman et al. (2010) Framework

Consumption

Within each sector, consumers demand a continuum of differentiated varieties. The aggregate consumption index is

$$Q = \left[\int_{j \in J} q(j)^{\beta} dj \right]^{1/\beta},$$

[12] For full details, we refer the reader to the Helpman et al. (2010) paper.

where j indexes varieties, J is the set of varieties within the sector, $q(j)$ denotes consumption of variety j, and $0 < \beta < 1$. The demand function for variety j is given by

$$q(j) = A^{1/(1-\beta)} p(j)^{-1/(1-\beta)},$$

where A is a sectoral demand shifter and $p(j)$ is the price of variety j.

Production

As in Melitz (2003), there is a competitive fringe of potential firms that can choose to enter the market by paying an entry cost $f_e > 0$. Once a firm incurs the sunk entry cost, it observes its idiosyncratic value of θ, a parameter that is related to its productivity and its optimal production structure (as discussed below). θ is drawn from a Pareto distribution with scale parameter θ_{min} and shape parameter z, i.e. $G_\theta(\theta) = 1 - (\theta_{min}/\theta)^z$ for $\theta \geq \theta_{min} > 0$ and $z > 2$.[13] Once firms observe θ, they decide whether to exit or produce. Production involves a fixed cost of $f_d > 0$ units of the numeraire. Since in equilibrium all firms with the same value of θ behave symmetrically, firms can be indexed by θ.

Firms produce using a Constant Elasticity of Substitution (CES) technology with two types of labor inputs: skilled and unskilled workers (indexed by s and r, respectively). A firm's output depends on its value of θ, as well as its choice of how many workers of each type to hire (h_s and h_r), and the average match-specific ability of these workers ($\overline{a}_s$ and $\overline{a}_r$). Specifically, the production function is

$$y = \left[(\theta \mu_s \overline{a}_s h_s^\gamma)^\nu + (\mu_r \overline{a}_r h_r^\gamma)^\nu \right]^{1/\nu}, \tag{3}$$

where $0 < \nu < \beta$, and μ_s and μ_r are aggregate skill-augmenting technology parameters.[14] For simplicity, we normalize $\mu_r = 1$. μ_s can therefore be interpreted in relative terms, as the relative aggregate skill-bias of technology. The parameter θ enters into the production function as a firm-specific skill-augmenting parameter. Firms that draw higher values of θ will be more productive overall (absolute advantage), but productivity will be particularly high for their skilled workers (comparative advantage). Hence, θ is related both to the productivity and to the skill-bias of production of each firm. The model therefore incorporates a link between firm productivity and technological skill bias, in line with the empirical evidence documented in Table 1.[15]

[13] The assumption that $z > 2$ ensures that the variance of θ is finite.

[14] The assumption that $\nu < \beta$ ensures that employment and wages of both types of workers are increasing in θ, in line with the empirical evidence presented above.

[15] While our empirical measure of productivity in Table 1 (revenue per worker) does not directly correspond to θ, it can be shown that, in equilibrium, the firm's revenue per worker is a strictly increasing function of

Search, Screening and Wage Bargaining

Labor markets are skill-specific and there is a fixed aggregate supply of workers of each type (an assumption that we relax in Section 4.4). The firm must pay a search cost of b_ℓ in order to be matched with n_ℓ workers, $\ell = \{s, r\}$.[16] Consistent with the empirical evidence, we assume that skilled workers are relatively scarce and hence command a higher search cost, i.e. $b_s > b_r$. Workers of a given skill type are ex-ante identical but, upon matching with a firm, draw match-specific abilities from a Pareto distribution with shape parameter k and scale parameter a_{min}: $G_a(a) = 1 - (a_{min}/a)^k$; $a \geq a_{min} > 0$ and $k > 1$.[17] Ability is not observable by the firm or the worker, but a screening technology is available. By paying a screening cost $c\widetilde{a}_\ell^\delta/\delta$, firms are able to identify whether a worker's match-specific ability is above or below an (endogenously chosen) cutoff $\widetilde{a}_\ell$, where $\ell = \{s, r\}$, $c > 0$, and $\delta > k$.[18]

Wages are determined through Stole & Zwiebel (1996a,b) bargaining, under conditions of symmetric information. Since the screening technology only reveals whether a worker's match-specific ability is above or below $\widetilde{a}_\ell$, but not the specific ability of any individual worker, the expected ability of all hired workers of a given type is the same, and equal to $\overline{a}_\ell$, the expected value of a conditional on being above the threshold $\widetilde{a}_\ell$. Therefore, all workers of a given type within a given firm receive the same wage.

4.2 Key Equilibrium Properties

Closed-form solutions can be obtained for the equilibrium values of firm-level employment, wages, revenue, and profits for operating firms. Here we highlight the key properties of interest for our purposes. Derivation details of the key equilibrium relationships are given in Appendix C.1.

Firm-Level Employment

Firm-level employment for unskilled workers is given by

$$h_r(\theta) = h_{dr}\left[1 + \varphi(\theta)\right]^{\left(\frac{\beta\Lambda}{\nu\Gamma} - 1\right)\left(1 - \frac{k}{\delta}\right)},$$

θ (see Appendix C.2).

[16] b_ℓ is determined endogenously by labor market tightness and is proportional to workers' expected income outside the sector.

[17] This distribution is assumed to be common across both types of workers.

[18] The assumption that $\delta > k$ is also needed in order to ensure that employment and wages of both types of workers are increasing in θ.

where

$$\varphi(\theta) = \mu_s^{\nu/\Lambda} \left(\frac{b_s}{b_r}\right)^{-\gamma\nu/\Lambda} \theta^{\nu/\Lambda}.$$

Here, $\Lambda \equiv 1 - \nu\gamma - \nu(1-\gamma k)/\delta > 0$, $\Gamma \equiv 1 - \beta\gamma - \beta(1-\gamma k)/\delta > 0$, and $\Lambda > \Gamma$ due to the assumption that $\nu < \beta$. The definition of h_{dr} is provided in Appendix Equation (C.11).

Employment of skilled workers for a firm of type θ is given by

$$h_s(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-k/\delta} h_r(\theta).$$

The firm's skilled worker share is therefore

$$\frac{h_s(\theta)}{h(\theta)} = \frac{b_r\varphi(\theta)^{1-k/\delta}}{b_s + b_r\varphi(\theta)^{1-k/\delta}},$$

where $h(\theta) = h_s(\theta) + h_r(\theta)$.

As shown in Appendix C.2, these equilibrium equations imply

$$\frac{\partial h_r(\theta)}{\partial\theta} > 0, \qquad \frac{\partial h_s(\theta)}{\partial\theta} > 0, \qquad \frac{\partial h_s(\theta)/h(\theta)}{\partial\theta} > 0. \tag{4}$$

The model therefore predicts that more productive firms (i.e. firms with a higher value of θ) will employ a larger number of both skilled and unskilled workers and, as a consequence, will be larger than less productive firms. More productive firms will also have a higher skilled employment share, implying that skilled workers disproportionately sort towards high-productivity firms. These predictions are in line with the motivating evidence presented in Table 1.

Firm-Level Wages

Firm-level wages for unskilled workers are

$$w_r(\theta) = w_{dr}\left[1+\varphi(\theta)\right]^{\left(\frac{\beta\Lambda}{\nu\Gamma}-1\right)\frac{k}{\delta}}. \tag{5}$$

The definition of w_{dr} is provided in Appendix Equation (C.17).

Wages for skilled workers are given by

$$w_s(\theta) = \frac{b_s}{b_r}\varphi(\theta)^{k/\delta} w_r(\theta). \tag{6}$$

As shown in Appendix C.2, these equilibrium wage equations imply:

$$\frac{\partial w_r(\theta)}{\partial \theta} > 0, \qquad \frac{\partial w_s(\theta)}{\partial \theta} > 0, \qquad \frac{\partial w_s(\theta)/w_r(\theta)}{\partial \theta} > 0. \tag{7}$$

The model therefore generates wage differences between firms conditional on worker skill, with more productive firms paying higher wages to workers of both types. Intuitively, the wage differentials arise in the model due to the complementarity between worker abilities and firm productivity, which gives an incentive for more productive firms to screen more intensively and choose a higher ability threshold. In equilibrium, wages are bargained down to the replacement cost of a worker, and given that more productive firms set higher hiring standards, their workers are costlier to replace and hence are paid a higher wage.

Note that both the match-specific heterogeneity and the screening technology are crucial elements (in addition to the search and matching frictions) in order to generate wage differences between firms for workers of a given skill type. If workers (within skill groups) were homogeneous, firms would have no incentive to screen, and wages would be bargained down to the replacement cost of a worker, which would simply be the search cost b_ℓ, which is common across firms for workers of a given type. Firms would be heterogeneous along the size margin (employment), but there would be no wage inequality between firms conditional on skill.[19] On the other hand, if screening were not feasible, then the average expected ability of workers across all firms would be common and equal to the average match-specific ability in the population. In this case, the bargaining process would lead to a common wage across firms for all workers and once again there would be no wage inequality.

Equation (7) also shows that the model predicts that the skill premium w_s/w_r varies across firms and is increasing in θ. This prediction is not strongly supported by the data. As Table 1 shows, more productive and larger establishments in Germany tend to pay higher wages to skilled and unskilled workers in similar proportions, and hence there is no strong systematic relationship between the establishment's skill premium and its productivity and size. This is likely a consequence of the institutional features of the German wage bargaining process.[20]

Productivity Threshold

Finally, as is standard in heterogeneous firm models, the presence of a fixed production cost implies that there is a zero-profit cutoff for productivity, θ_d, such that a firm that draws a

[19] Felbermayr et al. (2011) is an example of such a framework.

[20] In Germany, even though union agreements allow for different wage levels by worker skill, wage increases negotiated between unions and employer federations tend to be the same across worker types.

productivity below this threshold exits without producing. Appendix C.1.2 shows how this productivity threshold can be pinned down using the Zero-Cutoff Profit condition, which requires the firm at the cutoff θ_d to make zero profits (Equation C.23), along with the Free Entry condition, which states that the expected profits for a potential entrant should equal the fixed entry cost (Equation C.24).

Summary

To summarize, the cross-sectional predictions of the model are that firms that draw a higher value of θ are larger, have a higher skilled worker share, and pay higher wages, both because they hire a larger proportion of skilled workers (who earn higher average wages than unskilled workers), and because they pay higher wages to their workers (compared to less productive firms) conditional on skill type. These relationships are in line with the motivating empirical evidence presented in Table 1.

4.3 Impacts of Skill-Biased Technological Change

Following the literature, we model skill-biased technological change (SBTC) as an exogenous increase in μ_s, the aggregate skill-augmenting parameter for the skilled labor input in the production function in Equation (3). This shock generates an exogenous increase in the relative demand for skilled labor. Note that the shock to μ_s is an aggregate shock impacting all firms in the economy; however, as we show below, the impacts of this common shock are very heterogeneous across firms with different productivity levels. Further note that we focus here on the *relative* effects of SBTC – that is, the effects of SBTC on wages and employment of skilled vs unskilled workers in low vs high productivity firms – rather than the *absolute* effects of SBTC on overall wage and employment levels.

The key implications of an increase in μ_s are the following:[21]

Prediction 1: *Increased Skilled Wage Premium* – Skill-biased technological change increases the skilled wage premium within all firms, and in the aggregate.

Proof: *As shown in Appendix C.3:*

$$\frac{\partial \left[w_s(\theta)/w_r(\theta)\right]}{\partial \mu_s} > 0$$

[21]In what follows, we assume that the search costs b_s and b_r are not affected by technological change. The search costs are proportional to workers' expected income outside the sector (outside option). Helpman et al. (2010) discuss conditions under which the outside options can be assumed to be constant, even when there are shocks with aggregate implications (such as trade opening, in the setting analyzed in their paper).

Implications: As in traditional models with perfect competition and homogeneous firms, holding the supply of skilled workers constant, the rise in demand for skilled workers induced by SBTC leads to a rise in overall inequality due to an increasing between-skill component, i.e. a higher wage differential between skilled and unskilled workers. In our setting, given that more productive (higher wage) firms have a higher skilled share, the increased skilled wage premiums also lead to higher between-firm inequality in average wages, all else equal.

Prediction 2: *Selection* – SBTC increases the productivity threshold for production θ_d.

Proof: *See Appendix C.3.*

Implications: By increasing the productivity threshold θ_d, SBTC leads to the exit of firms at the bottom of the productivity distribution. Although this reduces the support of the distribution among operating firms, the variance of productivity among these firms increases. This is due to the fact that the distribution of productivity among operating firms is a truncated Pareto distribution with scale parameter θ_d and shape parameter z, and the variance of this distribution is increasing in the scale parameter θ_d.[22] Intuitively, with a Pareto distribution, the increase in θ_d entails the exit of a mass of relatively homogeneous unproductive firms. This mass is shifted towards the tail of the distribution, leading to an increase in the variance of productivity among the firms that remain in operation. While this change does not affect the variance of *log* productivity, log wages are not directly proportional to log productivity. Instead, they are a more complicated function of θ (see Equations (5) and (6)). Hence, the increase in the variance of productivity may lead to an increase in the variance of log wages among firms operating in the market, which would contribute to the rise in between-firm inequality.

Prediction 3: *Differential Employment Growth* – SBTC strengthens the cross-sectional association between employment and productivity.

Proof: *As shown in Appendix C.3:*

[22] The variance is given by $\frac{z\theta_d^2}{(z-1)^2(z-2)}$. It should be noted that an increase in the productivity threshold θ_d will not increase the variance of productivity among operating firms for all distributions of firm productivity. For example, if firm productivity were uniformly distributed, an increase in the productivity threshold θ_d would lower the variance of firm productivity among operating firms. The Pareto assumption, however, is standard in the literature, and is supported by empirical evidence (see e.g. Axtell, 2001; Corcos et al., 2012).

$$\frac{\partial\left(\frac{\partial h_r(\theta)}{\partial\theta}\right)}{\partial\mu_s} > 0 \qquad \text{and} \qquad \frac{\partial\left(\frac{\partial h_s(\theta)}{\partial\theta}\right)}{\partial\mu_s} > 0.$$

Implications: This prediction implies that more productive firms become disproportionately larger in terms of employment relative to less productive firms. SBTC therefore leads to increased employment concentration in more productive firms. This shift in employment of both types of workers towards more productive firms (which pay higher wages) leads to an increase in (worker-weighted) between-firm wage inequality (by skill and overall).

Prediction 4: *Increased Sorting and Segregation by Skill* – SBTC strengthens the cross-sectional association between productivity and skilled employment shares, provided that firms employ relatively more unskilled than skilled workers at baseline (the empirically relevant case).[23]

Proof: *As shown in Appendix C.3:*

$$\text{If} \quad \frac{h_s(\theta)}{h_r(\theta)} < 1, \quad \text{then} \quad \frac{\partial\left(\frac{\partial h_s(\theta)/h(\theta)}{\partial\theta}\right)}{\partial\mu_s} > 0.$$

Implications: This prediction implies that more productive firms will increase their skilled employment share by more than less productive firms. In consequence, firms within industries will become more heterogeneous in their skill mix as a result of SBTC, resulting in more segregation of workers by skill. Moreover, skilled (high-wage) workers will increasingly sort into more productive (and hence high-wage) firms. Both increased segregation and increased sorting will contribute to the overall increase in between-firm wage inequality.

Prediction 5: *Differential Wage Growth* – SBTC strengthens the cross-sectional association between productivity and wages conditional on worker skill.

Proof: *As shown in Appendix C.3:*

$$\frac{\partial\left(\frac{\partial w_r(\theta)}{\partial\theta}\right)}{\partial\mu_s} > 0 \qquad \text{and} \qquad \frac{\partial\left(\frac{\partial w_s(\theta)}{\partial\theta}\right)}{\partial\mu_s} > 0.$$

[23] More than 80% of workers in our sample are in workplaces that employ more unskilled than skilled workers.

Implications: As a result of SBTC, wages for both types of workers disproportionately increase within more productive firms relative to less productive firms. Thus, firm wage premiums – wages firms pay to workers of the same skill type – become more dispersed, leading to a further increase in wage inequality (overall and by skill) across firms.

To summarize, the model unambiguously predicts that skill-biased technological change leads to an increase in between-firm wage inequality. This operates through various distinct channels. The first is an extension to the heterogeneous firm setting of the channel highlighted by traditional models of SBTC with competitive markets and no firm heterogeneity; that is, a rise in the wage of skilled workers relative to unskilled workers. The other channels are novel to our setting; they are: differential employment growth, segregation and sorting, and differential within-firm wage growth. All of the channels compound each other in driving the increase in wage inequality between firms. Selective entry and exit of firms may additionally contribute to the rise in between-firm inequality.

4.4 Impacts of an Increase in the Supply of Skilled Workers

The evidence presented in Figure 1 – and in particular the fact that the skilled wage premium has remained stable over time in Germany – suggests that the rise in demand for skilled workers due to SBTC has been accompanied by a rise in the supply of workers of this type. In this section we consider whether a rise in the supply of skilled workers fully mitigates the impacts of SBTC on inequality – as would be the case in a representative firm framework with competitive labor markets.

We model the rise in the supply of skilled workers as an exogenous fall in the skilled worker search cost. Intuitively, when skilled workers become more abundant, it becomes easier for firms to find workers of this type, therefore reducing this type-specific search cost.[24] Proofs of all of the results discussed in this section are provided in Appendix C.4.

In the model, an increase in the supply of skilled workers mitigates the (within-firm) rise in the skill premium induced by SBTC, as in traditional models.[25] The implications

[24]Technically, the search cost is an endogenous variable which depends on labor market tightness. However, as Helpman et al. (2010) discuss, the equilibrium search cost is pinned down solely by expected income in the outside sector, which is exogenous. We can think of an increase in the economy-wide supply of skilled workers as reducing the wage of skilled workers in the outside sector (all else equal) and hence reducing the cost of searching for a skilled worker for firms in the differentiated sector.

[25]Note that, even though the model unambiguously predicts that an increase in the supply of skilled workers reduces the within-firm skill premium, it does not unambiguously predict that the *aggregate* skill premium will fall. This is because, as discussed below, a skilled labor supply shock also shifts employment toward more productive firms which, according to the model, pay higher skill premiums. This reallocation effect may therefore offset the (within-firm) decline in the skill premium. In our empirical setting, however,

regarding the other channels through which wage inequality may increase are, however, more nuanced. First, the supply of skilled workers further increases the productivity threshold for production θ_d, thus amplifying the increase in the variance of productivity among operating firms induced by the technology shock, which in turn may contribute to the rise in between-firm wage inequality.

Furthermore, the inequality-enhancing effects of technology that operate through differential employment growth and through increased sorting and segregation by skill are also amplified if the technological change shock is accompanied by an increase in the supply of skilled workers. Intuitively, the reduced cost of hiring skilled workers disproportionately benefits more productive firms (which employ relatively more workers of this type). These more productive firms therefore expand (in relative terms) and also further increase their skill specialization.

Finally, regarding the differential wage growth channel, an increase in the supply of skilled workers amplifies the prediction that between-firm wage inequality increases for unskilled workers, but (under reasonable parameter assumptions) dampens the prediction that between-firm wage inequality increases for skilled workers.

To summarize, a simultaneous expansion in the supply of skilled workers counteracts the rise in the relative wage of skilled workers, as in traditional models of SBTC, but generally exacerbates the other channels. Hence, between-firm (and overall) wage inequality may rise even if SBTC occurs alongside an increase in the supply of skilled workers. In consequence, the notion of a 'race' between technology and the supply of skills, which is present in traditional models of technological change based on representative firms and perfectly competitive labor markets, is no longer present in this richer type of model.

5 Empirical Evidence

In this section we return to the BEH and IABEP data in order to test the various predictions of the model, and to decompose the relative empirical importance of the different channels that it highlights. Note that, even though the model predicts that the premium paid by a given firm to skilled and unskilled workers is different, our empirical findings in Table 1 show that larger or more productive establishments pay higher wages to skilled and unskilled workers in similar proportions. Hence, we continue to consider a unique (rather than a skill-specific) wage premium for each establishment, as in Section 3. Put differently, we allow

the distinction between within-firm and aggregate (within-industry) changes in skill premiums is minor, given that there is no systematic relationship between the establishment's skill premium and its productivity and size, as shown in Table 1.

the skill premium to differ across industries, but we assume that it is common across all workplaces within an industry.

5.1 Associations over Time and Longitudinal Changes within Establishments

We begin by exploring the evolution over time of the cross-sectional associations between various establishment characteristics. The model predicts that, due to ongoing skill-biased technological change and the differential changes that it induces for workplaces with different productivity levels, we should observe a strengthening of the cross-sectional relationship between establishment productivity and size (Prediction 3), skilled worker share (Prediction 4), and wage conditional on worker type (Prediction 5). A simultaneous increase in the supply of skilled workers tends to amplify these predictions. To test whether these predictions hold in the data, we estimate the associations from Table 1 separately for each year, controlling for 3-digit industry fixed effects, thereby focusing once again on within-industry associations.

Figure 4 plots the coefficients from yearly regressions with the establishment's size, skill share and wage premium as dependent variables, using data from the IABEP and (log) productivity as the key regressor.[26] All of the associations have indeed become substantially stronger over our sample period. For example, as shown in Panel A, while in the early 1990s a 1% increase in the establishment's productivity was associated with an increase in establishment size of about 0.1%, the association had increased to more than 0.4% by 2010. Similarly, Panel B shows a strengthening of the relationship between productivity and skilled employment shares, which is indicative of increased sorting over time of skilled workers towards high-productivity establishments. Panel C shows that the coefficient from the regression of average establishment log wages on log labor productivity tripled from about 0.05 in the early 1990s to 0.15 by 2010. The association between (log) establishment productivity and the establishment wage premium, accounting for skills or detailed occupations, likewise nearly tripled over the time period (Panels D and E).

Figure 5 confirms these findings drawing on the larger BEH data and using establishment size as the key regressor of interest. The figure shows that the associations between establishment size and overall wages, as well as between establishment size and establishment wage premiums (though not between establishment size and the skilled share) increased over the sample period. For example, as Panel D shows, while a 1% increase in establishment size was associated with a 0.06% increase in the establishment wage premium (controlling

[26] In line with the empirical findings in Table 1, we assume that the establishment wage premium is the same for skilled and unskilled workers. Put differently, we assume that the skill premium is common across all workplaces within an industry.

for skills) in the early 1990s, this association steadily rose to nearly 0.09% by 2010.

Table 2 complements this evidence by showing estimates based on a set of regressions that consider changes within establishments over (non-overlapping) 5-year windows. Panel A regresses changes in various establishment outcomes (conditional on survival) on baseline establishment size, plus a set of fully interacted 3-digit industry and year fixed effects. The results show that surviving establishments that are larger within their industry at baseline exhibit a larger increase in labor productivity, a larger increase in the employment share of skilled workers, higher wage growth overall, and a larger increase in their wage premiums. In line with the predictions of the model, these results show that establishments that perform better at baseline (in terms of their size) pull away even further from other establishments in their industry (in terms of their productivity, skilled worker share and the wages they pay). If viewed through the lens of the model, SBTC therefore amplifies, rather than reduces, differences in productivity, skill usage and pay across establishments within industries. A simultaneous expansion in the supply of skilled workers may have exacerbated these differences across establishments.

Panel B provides further evidence of differential employment growth across establishments, considering also establishments that exit the market. This panel shows the results of a set of regressions that use within-establishment percentage changes in employment over 5-year windows as the dependent variable, and link these changes to various baseline establishment characteristics (controlling for a set of fully interacted 3-digit industry and year fixed effects). Column (1) uses establishment productivity as the regressor of interest and confirms that establishments that are more productive at baseline (within their 3-digit industry) grow significantly more than less productive establishments in the industry over subsequent years. The remaining columns show that establishments with initially higher skill shares and establishments that pay higher wages at baseline – overall and conditional on worker skill or occupation – also exhibit significantly larger employment growth. This evidence is consistent with the idea that SBTC (as well as a simultaneous increase in the supply of skilled workers) shifts employment toward more productive, higher wage establishments.

5.2 Segregation and Sorting

As shown above, larger establishments (which pay higher wages) have disproportionately increased their skilled worker employment shares. This suggests an increase in the sorting of skilled workers towards higher paying establishments, and an increase in worker clustering by skill. We verify this in Figure 6.

Panel A plots the within-industry variance of establishments' skilled employment shares over time, averaged across industries using either the contemporaneous or the 1990 industry structure (see Appendix B.6 for details). The figure shows a clear increase in the variance of skilled employment shares across establishments within industries. Thus, rather than having converged towards a more uniform mode of production, establishments have become increasingly heterogeneous in terms of the skill input mix that they use. Put differently, segregation by skill has increased across establishments within industries.

Panel B of Figure 6 shows the evolution of the within-industry co-variance between establishments' skilled employment shares and their wage premiums (conditional on skills), averaged once again across industries either using the contemporaneous or the 1990 industry structure (see Appendix B.7 for details). This co-variance also shows a clear positive trend over time: skilled (high-wage) workers increasingly sort into establishments that pay higher wage premiums.

This evidence, which our model rationalizes as being driven by skill-biased technological change and potentially exacerbated by a simultaneous increase in the supply of skilled workers, is consistent with the patterns documented by Card et al. (2013) and Song et al. (2019), which show that high-wage workers increasingly sort into high-wage firms and that high-wage workers are increasingly likely to work with each other.

5.3 Decompositions of Changes in Wage Inequality

In this section we perform a series of decompositions in order to assess the relative quantitative importance of the different channels highlighted by the model. We decompose the change in the within-industry between-establishment variance of log wages, denoted as $Var_{kt}(\overline{\ln w_{ft}})$ and defined as in Appendix Equation (B.6), focusing on non-overlapping 5-year windows in our data.

Selection. We begin by assessing the role of changes in the composition of operating establishments (i.e. selection). The model predicts that the composition of operating firms will become more positively selected, thereby leading to an increase in the variance of productivity among operating firms (as firm productivity is drawn from a Pareto distribution), and, potentially, to an increase in the variance of log wages (Prediction 2).

To quantify the importance of this channel, we first classify establishments as continuing, exiting or entering according to their status in periods $t-5$ and t. We can then decompose the change in between-establishment wage inequality in industry k between $t-5$ and t into a "selection effect" due to selective establishment entry and exit, and a change in the variance of establishment wages among continuing establishments:

$$\Delta Var_{kt}(\overline{\ln w}_{ft}) = \underbrace{Var_{kt} - Var^{con}_{kt} + Var^{con}_{kt-5} - Var_{kt-5}}_{\text{selection}} + \underbrace{\Delta Var^{con}_{kt}}_{\text{continuing establishments}}, \tag{8}$$

where for clarity $Var_{kt}(\overline{\ln w}_{ft}) = Var_{kt}$ and Var^{con}_{kt} is the variance of establishment wages among continuing establishments in industry k. This decomposition yields two components. The first component captures changes in between-establishment wage inequality due to selective establishment entry and exit. Specifically, the model predicts that in the base period $t-5$ the within-industry variance of productivity among continuing establishments exceeds the variance among all establishments, and hence the variance of establishment wages among continuing establishments (Var^{con}_{kt-5}) may be higher than among all establishments (Var_{kt-5}). The second component captures changes in between-establishment wage inequality among continuing establishments.

We compute the two components for each industry over each 5-year window in our data and average across industries using 1990 industry employment shares as weights. Panel A of Figure 7 presents the time-averaged value of each component (giving equal weight to all four time intervals). The figure highlights that while selective entry and exit have contributed to the rise in within-industry between-establishment wage inequality over time, this effect is small in magnitude, accounting for only 6% of the total increase. The bulk of the rise in between-establishment wage inequality is driven by continuing establishments.

Sorting and Segregation. Next, we further decompose the change in the within-industry between-establishment variance among continuing establishments, ΔVar^{con}_{kt}, to determine the role of changes in worker sorting and segregation (Prediction 4).

To begin with, note that the establishment wage can be written as the sum of the establishment wage premium and the establishment predicted wage, i.e.: $\overline{\ln w}_{ft} = \widetilde{\ln w}_{ft} + \widehat{\ln w}_{ft}$, where $\widetilde{\ln w}_{ft}$ is the establishment wage premium and $\widehat{\ln w}_{ft}$ is the establishment predicted wage, which in turn is a function of the establishment's skill mix and the skilled wage premium in the industry. Given this relationship, we can decompose the change in the wage variance among continuing firms in industry k as follows:

$$\Delta Var^{con}_{kt}(\overline{\ln w}_{ft}) = \underbrace{\Delta Var^{con}_{kt}(\widetilde{\ln w}_{ft})}_{\text{within-group}} + \underbrace{\Delta Var^{con}_{kt}(\widehat{\ln w}_{ft})}_{\text{segregation}} + \underbrace{2\Delta Cov^{con}_{kt}(\widetilde{\ln w}_{ft}, \widehat{\ln w}_{ft})}_{\text{sorting}}. \tag{9}$$

The first component, $\Delta Var^{con}_{kt}(\widetilde{\ln w}_{ft})$, captures changes in the variance of the wage premiums paid by continuing establishments conditional on worker skill (i.e. changes in the variance within skill groups). We displayed the evolution of this variance, averaged across industries using the 1990 industry structure, in Panel C of Figure 2 (the light-grey diamonds), with the modification that in the figure we considered all and not only continuing establishments.

The second component, the change in the variance of predicted establishment wages $\Delta Var^{con}_{kt}(\widehat{\ln w}_{ft})$, captures both changes in the skilled wage premium and changes in the segregation of worker types across establishments. To see this, note that the predicted wage of establishment f in industry k in year t is equal to the average wage of unskilled workers in the industry and year $(\overline{\ln w}^{r}_{kt})$, plus the share of skilled workers in the establishment (S_{ft}) multiplied by the industry-year-specific skilled wage premium $(SkillPrem_{kt})$; i.e. $\widehat{\ln w}_{ft} = \overline{\ln w}^{r}_{kt} + S_{ft} \cdot SkillPrem_{kt}$. Thus, the within-industry variance in predicted establishment wages among continuing establishments equals:

$$\begin{aligned} Var^{con}_{kt}(\widehat{\ln w}_{ft}) &= Var^{con}_{kt}(\overline{\ln w}^{r}_{kt} + S_{ft} \cdot SkillPrem_{kt}) \\ &= SkillPrem^{2}_{kt} \cdot Var^{con}_{kt}(S_{ft}). \end{aligned}$$

Since the skilled wage premium remained roughly constant over time (Panel B of Figure 1), whereas the variance of establishments' skilled employment shares increased over time (Panel A of Figure 6), we expect any changes in the within-industry variance of predicted establishment wages to primarily reflect changes in the variance of establishments' skilled employment shares. This observation motivates us to refer to this term as changes in worker segregation by skill.

In turn, the third component in Equation (9), $\Delta Cov^{con}_{kt}(\widetilde{\ln w}_{ft}, \widehat{\ln w}_{ft})$, captures the increased sorting of skilled workers into establishments paying higher wage premiums. To see this, note that the co-variance between establishments' predicted wages and their wage premiums equals:

$$\begin{aligned} Cov^{con}_{kt}(\widehat{\ln w}_{ft}, \widetilde{\ln w}_{ft}) &= Cov^{con}_{kt}(\overline{\ln w}^{r}_{kt} + S_{ft} \cdot SkillPrem_{kt}, \widetilde{\ln w}_{ft}) \\ &= SkillPrem_{kt} \cdot Cov^{con}_{kt}(S_{ft}, \widetilde{\ln w}_{ft}). \end{aligned}$$

Based again on the finding of a stable skilled wage premium, changes in this term will be primarily driven by changes in the co-variance between establishment skill shares and establishment premiums (Panel B of Figure 6), i.e. by the increased sorting of skilled workers to establishments that pay higher wage premiums.

The results of this decomposition are presented in Panel B of Figure 7, where we once again average across industries using the 1990 industry structure as weights and give equal weight to each 5-year period. Even though we distinguish between two skill groups only, increased sorting of skilled workers to establishments paying higher wage premiums (the third component in Equation 9) can account for about 12% of the overall increase in within-industry between-establishment wage inequality among continuing establishments. While dispersion in skill usage or segregation has increased over time across establishments within the same industry (see Panel A of Figure 6), its contribution to the overall increase in between-establishment wage inequality is minor (the second component in Equation 9). Not surprisingly, given that we only distinguish between two skill groups, the within-group component (i.e., changes in the variance of establishment wage premiums) accounts for the majority (86.1%) of the change in the within-industry variance among continuing establishments.

In Appendix Figure A.1, we repeat the exercise distinguishing between 317 occupations, rather than two skill groups. As expected, increased segregation (or dispersion in the occupational structure) across establishments (i.e., the second component in Equation 9) and increased sorting of workers in high-paying occupations into establishments paying high establishment premiums (i.e., the third component in Equation 9) become quantitatively more important, accounting for about 20% and 25% of the overall increase in the within-industry wage variance among continuing establishments, respectively. The change in the variance of establishment wage premiums, however, remains the dominant component also when considering this detailed occupational level.

Differential Employment Growth vs Differential Wage Growth. In a final step, our goal is to gauge the importance of differential employment growth for the increase in between-establishment wage inequality. In line with the prediction of the model, we have documented that the association between establishment productivity and size has become stronger over time (Panel A of Figures 4 and 5) and that establishments that were initially more productive (and those that paid higher wages) grow at faster rates (Panel B of Table 2). These differential employment growth rates will result in an increase in wage inequality across establishments, even if establishment wage premiums had remained unchanged.

To assess the importance of this channel, we focus on the change in the variance of the establishment wage premiums, $\Delta Var_{kt}^{con}(\widetilde{\ln w}_{ft})$, and decompose this variance into the role that is due to differential employment growth (Prediction 3), and a residual component that is attributable to differential changes in wage premiums within establishments (Prediction 5). This final decomposition is given by:

$$\Delta Var_{kt}^{con}(\widetilde{\ln w}_{ft}) = \underbrace{\sum_{f \in f_{kt}^{con}} \left[\Delta\left(\frac{n_{ft}}{n_{kt}^{con}}\right)\right]\left(\widetilde{\ln w}_{ft-5} - \overline{\widetilde{\ln w}}_{kt-5}^{con}\right)^2}_{\text{differential employment growth}} + \underbrace{\sum_{f \in f_{kt}^{con}} \frac{n_{ft}}{n_{kt}^{con}} \Delta\left(\widetilde{\ln w}_{ft} - \overline{\widetilde{\ln w}}_{kt}^{con}\right)^2}_{\text{residual (differential wage growth)}}, \quad (10)$$

where f_{kt}^{con} is the set of continuing firms in industry k at time t, n_{kt}^{con} denotes total employment among continuing establishments in industry k and time t and $\overline{\widetilde{\ln w}}_{kt}^{con}$ denotes the average wage premium in industry k in period t among continuing establishments. The first component of the decomposition therefore shows the changes in the variance that arise solely from changes in establishment size, holding establishment wage premiums at $t-5$ levels.

Panel C of Figure 7 shows the results from this decomposition. The figure shows that differential employment growth is an important driver behind the increase in the variance of establishment wage premiums among continuing establishments, accounting for around half of the increase. Thus, the fact that establishments grow at different rates accounts for an important share of the increase in wage inequality, even abstracting from any changes in wages within establishments. The residual component is also important, indicating that the increased dispersion in establishment wage premiums is also quantitatively relevant. Panel B of Appendix Figure A.1 presents the analogous results based on the establishment wage premiums that control for detailed occupations, rather than skills, and also shows a very important role for differential employment growth.

Overall, we can conclude that all of the channels highlighted by the model have contributed to the rise in wage inequality between establishments within industries, with the exception of changes in the skilled wage premium – the sole channel that emerges from traditional models of skill-biased technological change – given that this has remained stable over time, likely due to the rising supply of skilled workers. While changes in segregation play only a minor role, changes in the composition of operating establishments, sorting of skilled workers to high-wage establishments, differential employment growth, and differential within-establishment wage growth are all quantitatively important, with the latter two being of primary importance.

5.4 Technology Adoption: Industry-Level Analysis

As a final support of the implications of the model, we leverage variation in direct measures of technology adoption at the industry level. Specifically, we analyze whether industries with more technology adoption have experienced larger increases in overall between-establishment wage inequality, in the variance of establishments' skilled employment shares and their wage premiums, and in the sorting of skilled workers to high-wage establishments.

We first consider industry-level variation in the change of the skilled worker share between 1990 and 2010. If we think of changes in the supply of skilled workers as an aggregate common shock impacting all industries, we can interpret differential changes in industry-level skilled employment shares as being driven by differential exposure to skill-biased technological change. For simplicity, we divide industries into two groups, based on whether they experience above-median or below-median increases in the skilled employment share over the entire period.[27]

Panel A of Figure 8 shows the evolution over time of the overall variance of establishment (log) wages for these two groups of industries. In line with the theoretical model, we find that industries that experience larger increases in their skilled employment shares also experience a stronger increase in wage inequality between establishments. Panels B and C further highlight that the variance in establishment wage premiums, adjusting for the skill and occupation composition in the establishment, increased more in industries that experienced a larger overall increase in the skilled employment share. As shown in Panel D, these industries also show a stronger increase in the between-establishment variance of the employment share of skilled workers. Hence, establishments have become increasingly heterogeneous in their skill mix particularly in industries experiencing a larger overall increase in skilled employment. Finally, Panel E provides evidence of larger increases in the sorting of skilled workers to high wage premium establishments in industries characterized by larger increases in their skilled employment shares.

Figure 9 exploits a more direct measure of technology exposure based on the change in robots per worker within industries over the 1993-2010 time period, using data from the International Federation of Robotics. Once again we divide industries into two groups, according to whether they experience above or below median changes during this time period.

Panel A first confirms that we can think of robot adoption as biased technological change: Industries with above-median robot adoption experience a much larger increase in their skilled employment shares. Panels B to D further show that these industries also experience larger increases in the variance of average establishment (log) wages and establishment wage premiums that adjust for the skill and occupation structure in the establishment. The remaining panels show that establishments are becoming increasingly heterogeneous in terms of their skill mix particularly in industries with above-median robot adoption (Panel E), and that the sorting of skilled workers into high-wage establishments is also particularly pronounced in these industries (Panel F).

[27]The median is computed based on the employment distribution across industries in 1990.

Finally, Figure 10 shows that we obtain broadly consistent results if we use a measure of technology adoption based on the industry's change in ICT capital stock per worker between 1991 and 2007 from the EUKLEMS data.[28] Industries with more technology adoption tend to experience larger increases in their skill shares (Panel A), larger increases in the dispersion of average establishment (log) wages (Panel B) and establishment wage premiums (Panels C and D), larger increases in the dispersion of establishments' skill input mix (Panel E), and more sorting of skilled workers towards high wage establishments (Panel F).

In line with the predictions from the theoretical model, these results show that increases in between-establishment wage inequality, segregation and sorting are generally stronger in industries more affected by SBTC. In principle, however, SBTC-intensive industries may also have been more exposed to other shocks, such as trade or offshoring, which may also have impacted these between-establishment trends within industries. To rule out this possibility as much as possible, in Appendix Tables A.2 and A.3 we analyze whether the gaps that we observe between more and less technology-exposed industries are robust to controlling for industry-level measures of trade exposure and offshorability.

The first column of Tables A.2 and A.3 reports our baseline estimates, obtained by regressing the industry's change over time in the outcome variable of interest (inequality, segregation or sorting) on an indicator variable for whether the industry is above or below the median in terms of technology adoption. The table reports the estimated coefficient on this dummy variable from regressions that use the change between 1990 and 2000, and the change between 1990 and 2010, respectively. These estimates correspond to the difference between the black dots and the grey diamonds for the years 2000 and 2010 in Figures 8 9, respectively. They confirm the excess increase in the various components of between-establishment wage inequality in industries more exposed to technological change, relative to those that are less exposed.

In Column (2), we add controls for the industry's exposure to trade with China and Eastern Europe over the 1990 to 2010 period. Specifically, we include the industry-level change between 1990 and 2010 in exports and imports per worker (to and from these countries) as additional control variables.[29] Adding these control variables has little impact on our estimates. In Column (3), we instead control for the extent to which each industry is susceptible to offshoring. We do so by drawing on data provided by Goos et al. (2014) on occupation-level offshorability, which we aggregate up to the industry level using each industry's 1990 occupational structure. Conditioning on the industry's offshorability likewise has almost no

[28]Note that EUKLEMS data is only available at a more aggregated industry level and hence this analysis is based on 48 2-digit industries.

[29]Trade data is obtained from the UN Comtrade Database.

effect on our estimates. Finally, in Column (4), where we jointly control for the industry's trade exposure and offshorability, the estimates still remain largely unchanged.

While these findings do not rule out the possibility that shocks other than SBTC have also contributed to the rise in wage inequality, they corroborate the importance of SBTC as a driver of between-establishment inequality, segregation and sorting, even conditional on other shocks at the industry level. SBTC is therefore an important driver of wage inequality not only across workers with different skill levels, but also across workplaces within an industry.

6 Conclusions

In this paper, we show that skill-biased technological change is an important driver of the rise in between-establishment wage inequality in Germany between 1990 and 2010. While a large literature has considered the role of skill-biased technological change for wage inequality, it has focused on representative firm frameworks with perfectly competitive labor markets and has hence had implications solely in terms of wage differentials *between* workers with different skill levels. Empirically, however, a major component of the increase in wage inequality is observed *within* skill groups, across establishments within industries.

By embedding a skill-biased technological change shock within a rich, yet tractable heterogeneous firm framework, we show that this type of shock will lead to heterogeneous responses at the firm level, thereby generating a rise in between-firm wage inequality. Using detailed administrative social security data from Germany, we document a number of novel empirical patterns at the establishment level, and show that these patterns are in line with the predictions of the model. The model highlights that the rise in inequality occurs due to endogenous changes in worker sorting, establishment size and establishment wages paid to the same worker type, possibly compounded by endogenous changes in the composition of operating firms. We find that all of these channels are empirically relevant. We also provide evidence that the key workplace-level patterns that we identify as being driven by the technological change shock are indeed more pronounced within industries that have experienced stronger rates of technology adoption, even when controlling for trade shocks and offshorability at the industry level.

The literature on technological change has long thought about increases in educational attainment as being a useful tool to offset the rise in inequality (Tinbergen, 1974, 1975; Acemoglu & Autor, 2011). In a homogeneous firm setting with competitive labor markets, a rise in the supply of skilled workers will offset (if large enough) the rise in the skilled wage premium and thus the rise in (between-group) inequality that arises due to skill-biased technological change. When moving away from this competitive homogeneous firm framework, however, the idea that there is a simple "race" between technology and the supply of skills is no longer valid. We show that in our richer heterogeneous firm framework, an increase in educational attainment may mitigate the rise in the skilled wage premium (as is indeed observed in the German case over our sample period), but it actually compounds the effects of technological change on (within-group) wage inequality that operate through some of the other channels highlighted by the model. Hence, expanding educational attainment may not be sufficient to dampen the rise in wage inequality induced by skill-biased technological change.

Overall, our results point to the importance of moving beyond the traditional representative firm setting with competitive labor markets when considering the impact of aggregate shocks such as technological change. While the literature has generally thought about the individual-level impacts of skill-biased technological change as being related to the skills that individuals possess or the tasks that they perform (e.g. Cortes, 2016; Blien et al., 2021), our findings indicate that the type of firm that individuals are matched to is at least as important: Unskilled workers employed in low-productivity firms lose out not only relative to skilled workers in these firms, but also relative to unskilled workers in high-productivity firms. Understanding what type of policies can mitigate the negative impacts of technological change on some groups of workers, within the context of a more realistic environment with heterogeneous firms and various market frictions such as the ones considered in this paper, remains a crucial direction for future work.

References

Acemoglu, D. & Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. *Handbook of Labor Economics*, 4, 1043–1171.

Acemoglu, D., Lelarge, C., & Restrepo, P. (2020). Competing with robots: Firm-level evidence from France. *AEA Papers and Proceedings*, 110, 383–88.

Acemoglu, D. & Restrepo, P. (2020). Robots and jobs: Evidence from US labor markets. *Journal of Political Economy*, 128(6), 2188–2244.

Aghion, P., Antonin, C., Bunel, S., & Jaravel, X. (2020). What are the labor and product market effects of automation? New evidence from France. *Working Paper*.

Akerman, A., Gaarder, I., & Mogstad, M. (2015). The skill complementarity of broadband internet. *The Quarterly Journal of Economics*, 130(4), 1781–1824.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., & Van Reenen, J. (2017). Concentrating on the fall of the labor share. *American Economic Review: Papers & Proceedings*, 107(5), 180–185.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., & Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135(2), 645–709.

Autor, D. H., Dorn, D., & Hanson, G. H. (2015). Untangling trade and technology: Evidence from local labour markets. *The Economic Journal*, 125(584), 621–646.

Autor, D. H., Katz, L. F., & Kearney, M. S. (2006). The polarization of the US labor market. *The American Economic Review*, 96(2), 189–194.

Autor, D. H., Katz, L. F., & Krueger, A. B. (1998). Computing inequality: Have computers changed the labor market? *The Quarterly Journal of Economics*, 113(4), 1169–1214.

Autor, D. H., Levy, F., & Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics*, 118(4), 1279–1333.

Axtell, R. L. (2001). Zipf distribution of U.S. firm sizes. *Science*, 293(5536), 1818–1820.

Azar, J., Marinescu, I., Steinbaum, M., & Taska, B. (2020a). Concentration in US labor markets: Evidence from online vacancy data. *Labour Economics*, 66, 101886.

Azar, J., Marinescu, I. E., & Steinbaum, M. (2020b). Labor market concentration. *Journal of Human Resources*, Forthcoming.

Bajgar, M., Berlingieri, G., Calligaris, S., Criscuolo, C., & Timmis, J. (2019). Industry concentration in Europe and North America. *OECD Productivity Working Papers, No. 18*.

Barth, E., Bryson, A., Davis, J. C., & Freeman, R. (2016). It's where you work: Increases in the dispersion of earnings across establishments and individuals in the United States. *Journal of Labor Economics*, 34(S2), S67–S97.

Bessen, J., Goos, M., Salomons, A., & van den Berge, W. (2020). Firm-level automation: Evidence from the Netherlands. *AEA Papers and Proceedings*, 110, 389–393.

Bhaskar, V., Manning, A., & To, T. (2002). Oligopsony and monopsonistic competition in labor markets. *The Journal of Economic Perspectives*, 16(2), 155–174.

Blien, U., Dauth, W., & Roth, D. H. (2021). Occupational routine intensity and the costs of job loss: Evidence from mass layoffs. *Labour Economics*, 68, 101953.

Bonfiglioli, A., Crinò, R., Fadinger, H., & Gancia, G. (2020). Robot imports and firm-level outcomes. *Working Paper*.

Card, D., Cardoso, A. R., Heining, J., & Kline, P. (2018). Firms and labor market inequality: Evidence and some theory. *Journal of Labor Economics*, 36(S1), S13–S70.

Card, D., Heining, J., & Kline, P. (2013). Workplace heterogeneity and the rise of West German wage inequality. *The Quarterly Journal of Economics*, 128(3), 967–1015.

Corcos, G., Gatto, M. D., Mion, G., & Ottaviano, G. I. (2012). Productivity and firm selection: Quantifying the 'new' gains from trade. *The Economic Journal*, 122, 754–798.

Cortes, G. M. (2016). Where have the middle-wage workers gone? A study of polarization using panel data. *Journal of Labor Economics*, 34(1), 63–105.

Cortes, G. M. & Salvatori, A. (2019). Delving into the demand side: Changes in workplace specialization and job polarization. *Labour Economics*, 57, 164–176.

Cortes, G. M. & Tschopp, J. (2020). Rising concentration and wage inequality. *IZA Discussion Paper No. 13557*.

Criscuolo, C., Hijzen, A., Schwellnus, C., Barth, E., Chen, W.-H., Fabling, R., Fialho, P., Grabska, K., Kambayashi, R., Leidecker, T., Skans, O. N., Riom, C., Roth, D., Stadler, B., Upward, R., & Zwysen, W. (2020). Workforce composition, productivity and pay: The role of firms in wage inequality. *OECD Social, Employment and Migration Working Papers No. 241*.

Dauth, W., Findeisen, S., Suedekum, J., & Woessner, N. (2021). The adjustment of labor markets to robots. *Journal of the European Economic Association*, 10.1093/jeea/jvab012.

Diamond, P. A. (1982a). Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5), 881–894.

Diamond, P. A. (1982b). Wage determination and efficiency in search equilibrium. *The Review of Economic Studies*, 49(2), 217–227.

Dustmann, C., Fitzenberger, B., Schönberg, U., & Spitz-Oener, A. (2014). From sick man of Europe to economic superstar: Germany's resurgent economy. *Journal of Economic Perspectives*, 28(1), 167–88.

Dustmann, C., Ludsteck, J., & Schönberg, U. (2009). Revisiting the German wage structure. *Quarterly Journal of Economics*, 124(2), 843–881.

Felbermayr, G., Prat, J., & Schmerer, H.-J. (2011). Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity. *Journal of Economic Theory*, 146(1), 39–73.

Fitzenberger, B. & Seidlitz, A. (2020). The 2011 break in the part-time indicator and the evolution of wage inequality in Germany. *Journal for Labour Market Research*, 54(1), 807–836.

Goldin, C. & Katz, L. F. (2008). Long-run changes in the wage structure: Narrowing, widening, polarizing. *Brookings Papers on Economic Activity*, 2007(2), 135–165.

Goos, M., Manning, A., & Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review*, 104(8), 2509–2526.

Graetz, G. & Michaels, G. (2018). Robots at work. *The Review of Economics and Statistics*, 100(5), 753–768.

Haanwinckel, D. (2020). Supply, demand, institutions, and firms: A theory of labor market sorting and the wage distribution. *Working Paper*.

Haltiwanger, J., Hyatt, H., & Spletzer, J. R. (2021). Industries, mega firms, and increasing inequality. *NBER Wage Dynamics in the 21st Century Conference Working Paper*.

Helpman, E., Itskhoki, O., Muendler, M.-A., & Redding, S. J. (2017). Trade and inequality: From theory to estimation. *Review of Economic Studies*, 84(1), 357–405.

Helpman, E., Itskhoki, O., & Redding, S. (2010). Inequality and unemployment in a global economy. *Econometrica*, 78(4), 1239–1283.

Jaimovich, N. & Siu, H. E. (2020). Job polarization and jobless recoveries. *The Review of Economics and Statistics*, 102(1), 129–147.

Katz, L. F. & Murphy, K. M. (1992). Changes in relative wages, 1963–1987: Supply and demand factors. *The Quarterly Journal of Economics*, 107(1), 35.

Koch, M., Manuylov, I., & Smolka, M. (2021). Robots and firms. *The Economic Journal*, 131, 2553–2584.

Kramarz, F., Lollivier, S., & Pele, L.-P. (1996). Wage inequalities and firm-specific compensation policies in France. *Annales d'Economie et de Statistique*, 41/42, 369–386.

Lindenlaub, I. (2017). Sorting multidimensional types: Theory and application. *The Review of Economic Studies*, 84(2), 718–789.

Lindner, A., Muraközy, B., Reizer, B., & Schreiner, R. (2021). Firm-level technological change and skill demand. *Working Paper*.

Machin, S. & Van Reenen, J. (1998). Technology and changes in skill structure: Evidence from seven OECD countries. *The Quarterly Journal of Economics*, 113(4), 1215–1244.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695–1725.

Michaels, G., Natraj, A., & Van Reenen, J. (2014). Has ICT polarized skill demand? Evidence from eleven countries over twenty-five years. *Review of Economics and Statistics*, 96(1), 60–77.

Mortensen, D. T. & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3), 397–415.

Mueller, H. M., Ouimet, P. P., & Simintzi, E. (2017). Wage inequality and firm growth. *American Economic Review: Papers & Proceedings*, 107(5), 379–83.

Rinz, K. (2020). Labor market concentration, earnings, and inequality. *Journal of Human Resources*, Forthcoming.

Song, J., Price, D. J., Guvenen, F., Bloom, N., & von Wachter, T. (2019). Firming up inequality. *The Quarterly Journal of Economics*, 134(1), 1–50.

Stole, L. A. & Zwiebel, J. (1996a). Intra-firm bargaining under non-binding contracts. *The Review of Economic Studies*, 63(3), 375–410.

Stole, L. A. & Zwiebel, J. (1996b). Organizational design and technology choice under intrafirm bargaining. *The American Economic Review*, 86(1), 195–222.

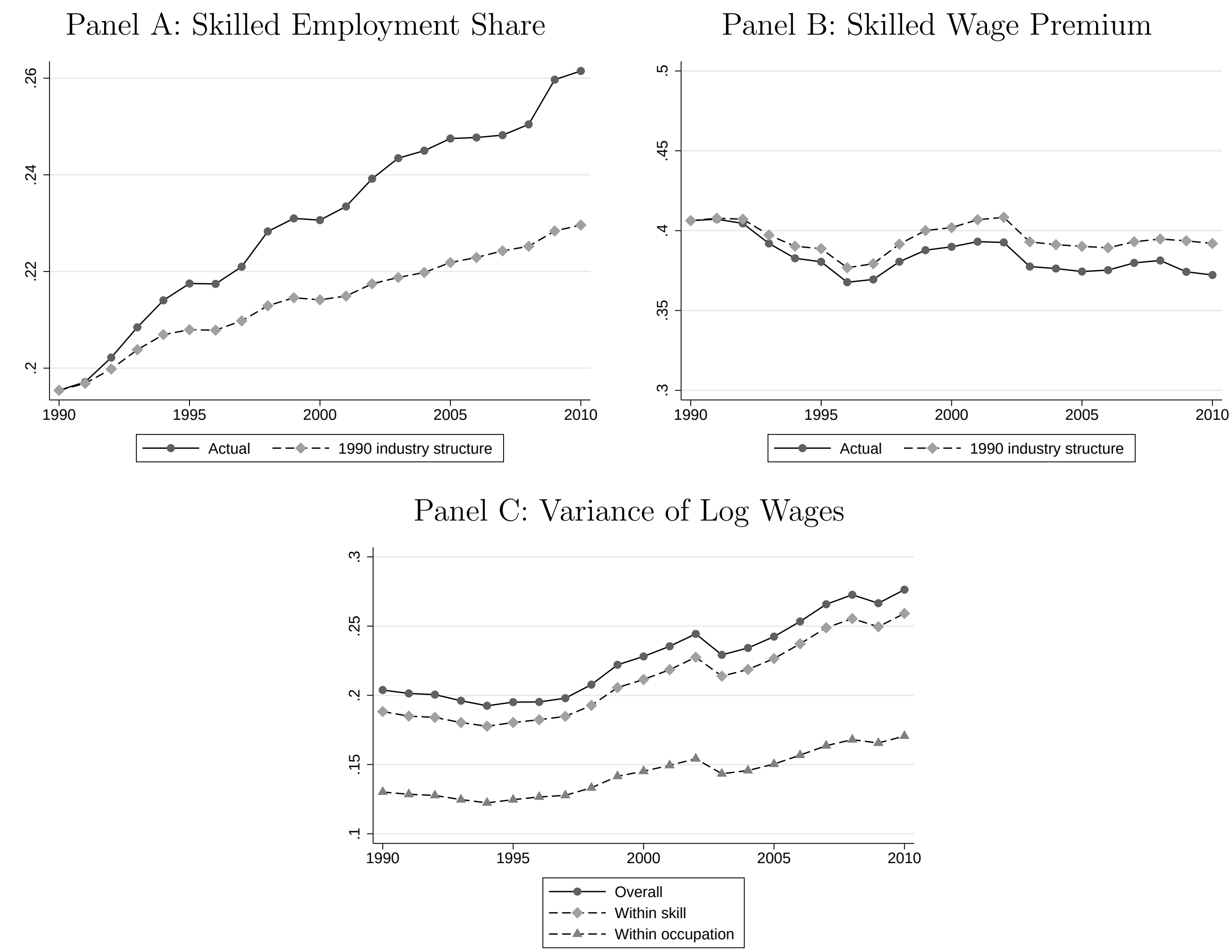
Tinbergen, J. (1974). Substitution of graduate by other labor. *Kyklos*, 27(2), 217–226.

Tinbergen, J. (1975). *Income Differences: Recent Research*. North-Holland Publishing Company.

Webber, D. A. (2015). Firm market power and the earnings distribution. *Labour Economics*, 35, 123 – 134.

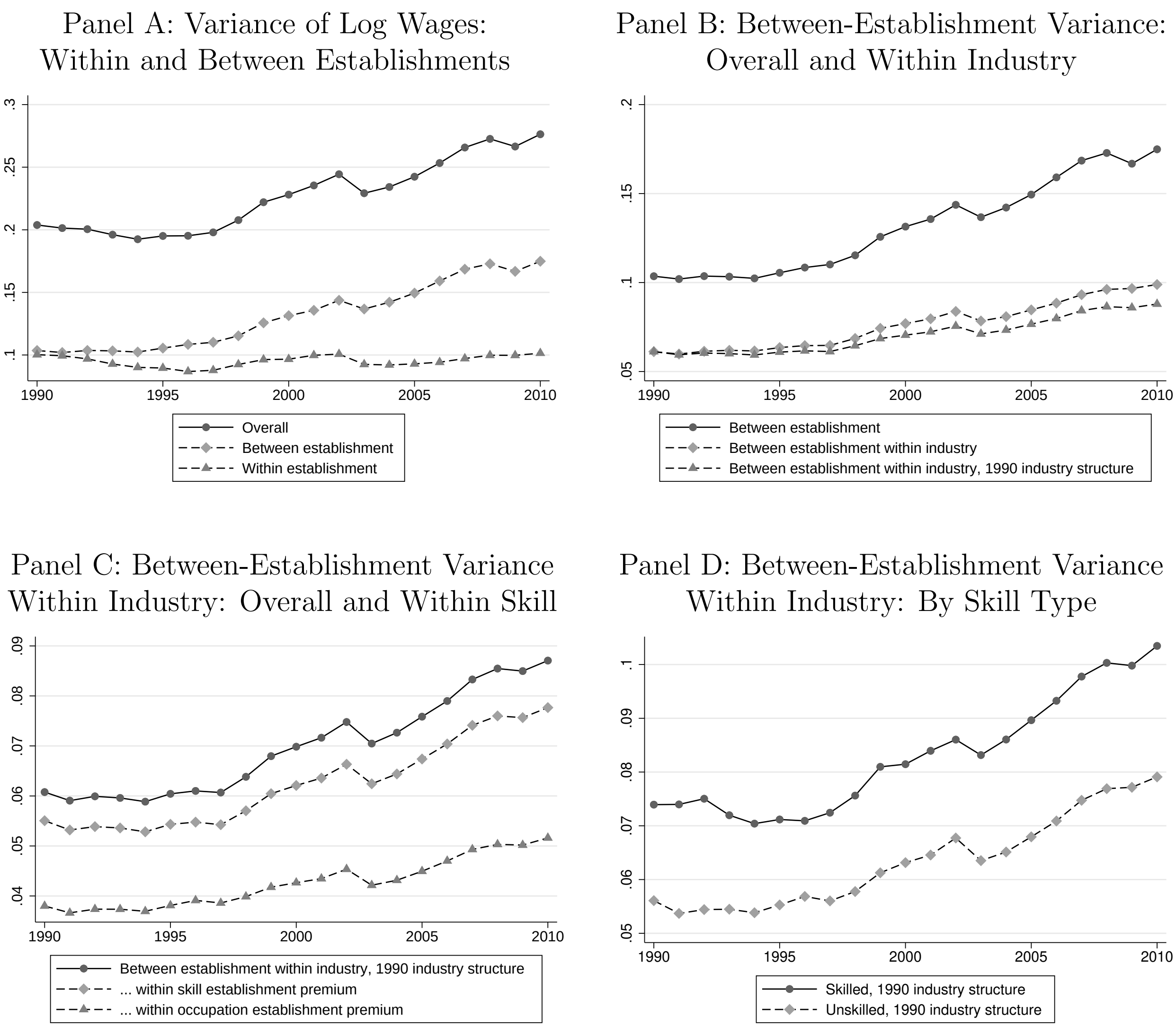
Wilmers, N. & Aeppli, C. (2021). Consolidated advantage: New organizational dynamics of wage inequality. *Washington Center for Equitable Growth Working Paper*.

Figure 1: Evolution of Skill-Related Labor Market Outcomes



Note: Panel A shows the evolution of the share of skilled workers in overall employment in Germany between 1990 and 2010 based on data from the Beschäftigtenhistorik (BEH). The solid line uses the contemporaneous industry structure in each year, while the dashed line fixes the industry composition using 1990 employment shares and therefore captures only changes in the skill composition of employment within industries (see Appendix B.1). Panel B shows the evolution of the skilled wage premium, calculated as the difference between the average log wage of full-time skilled and unskilled workers separately for each industry and year, and then averaged across industries using the actual or 1990 industry structure (see Appendix B.2). Panel C displays the evolution of the overall, within-skill (based on two skill groups) and within-occupation (based on 317 occupations) variance of individual log wages (see Appendix B.3). The mapping of occupations to skill categories is presented in Appendix Table A.1.

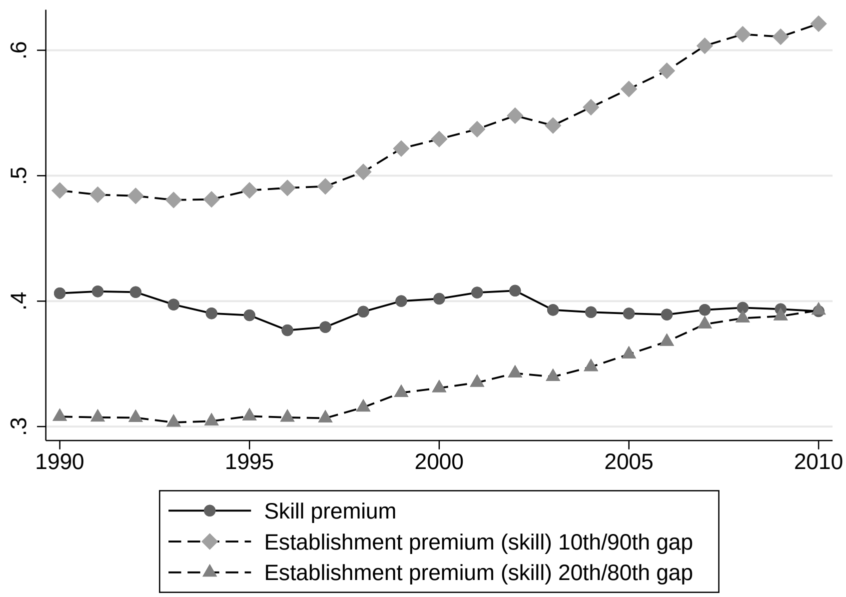
Figure 2: Evolution of the Variance of Log Wages



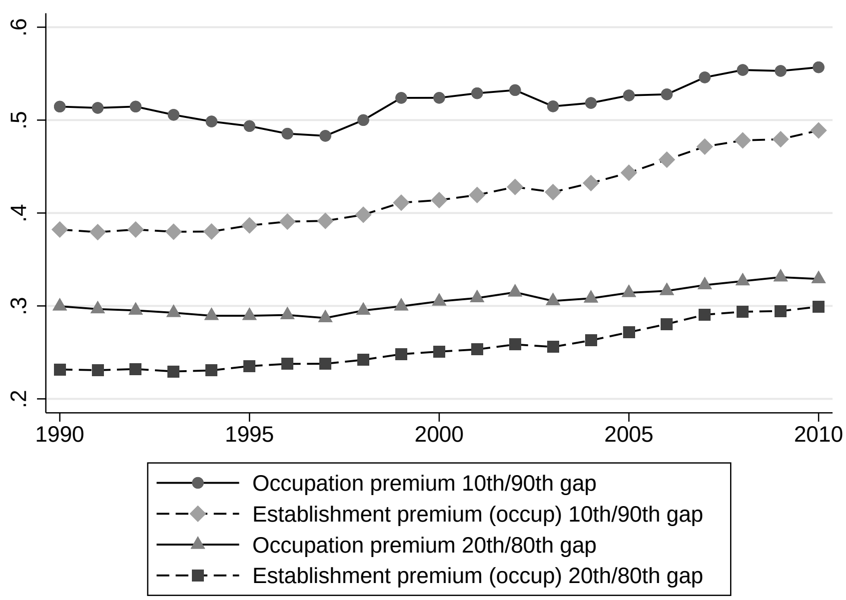
Note: Panel A displays the evolution of the overall variance of individual log wages and its within- and between-establishment components, as described in Equation (1). Panel B displays the overall and the within-industry between-establishment wage variance, computed using the actual and 1990 industry structure, as shown in Equation (2) and Appendix B.4. Panel C displays the within-industry variance in establishment wage premiums based on either two broad skill groups or 317 detailed occupation groups, averaged across industries using the 1990 industry structure (see Appendix B.5). Establishment wage premiums adjust for the skill and occupation composition of the establishment, and are computed as the establishment average of the residual of an individual wage regression, estimated separately for each year, that controls for skill groups (occupations) interacted with 3-digit industry fixed effects. Panel D displays the within-industry between-establishment wage variance for skilled and unskilled workers using the 1990 industry structure.

Figure 3: Establishment vs Skill and Occupation Premiums

Panel A: Establishment Premium Gaps (Two Skill Groups) and Skilled Wage Premium

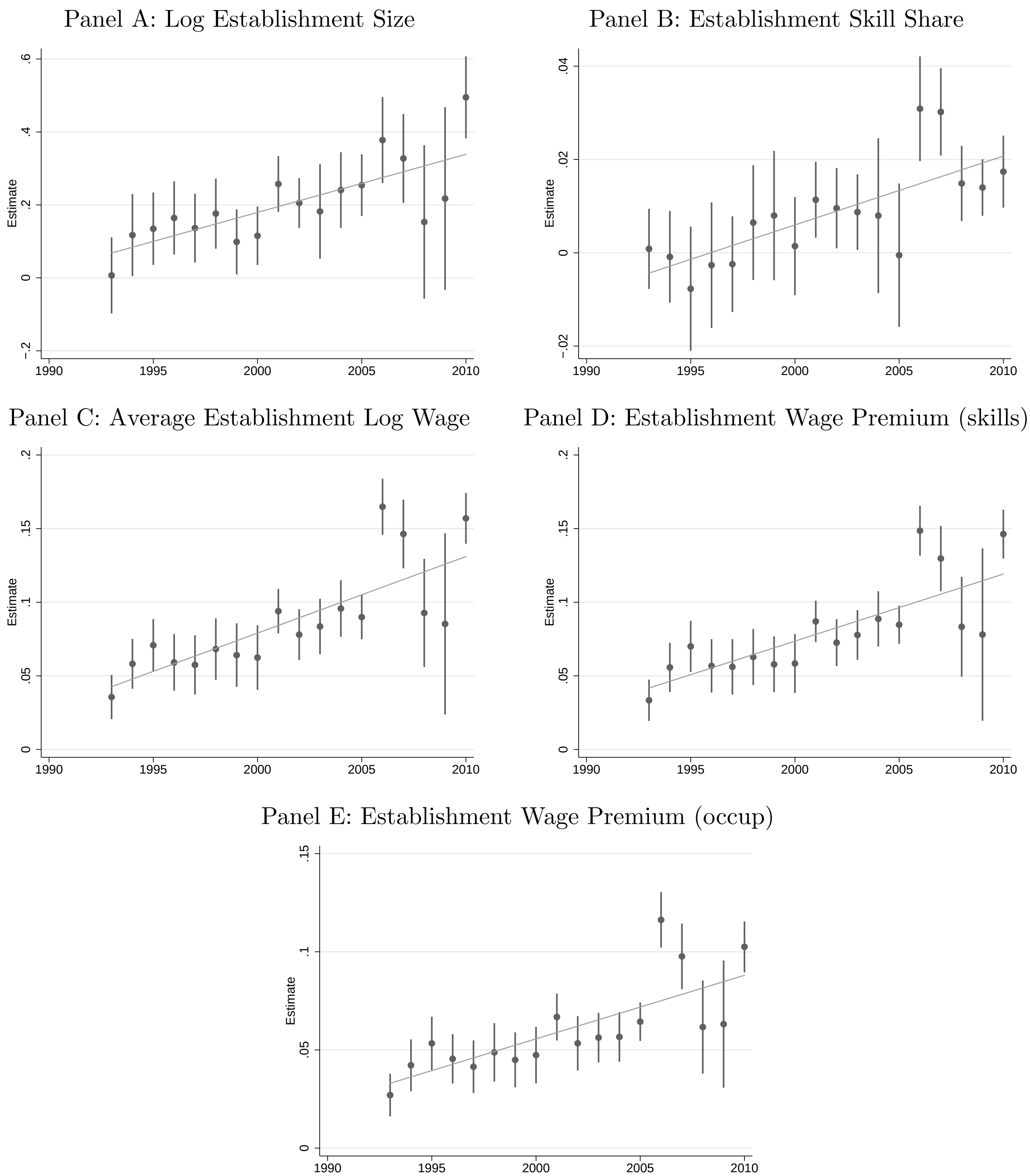


Panel B: Establishment Premium Gaps (317 Occupations) and Occupation Wage Premiums



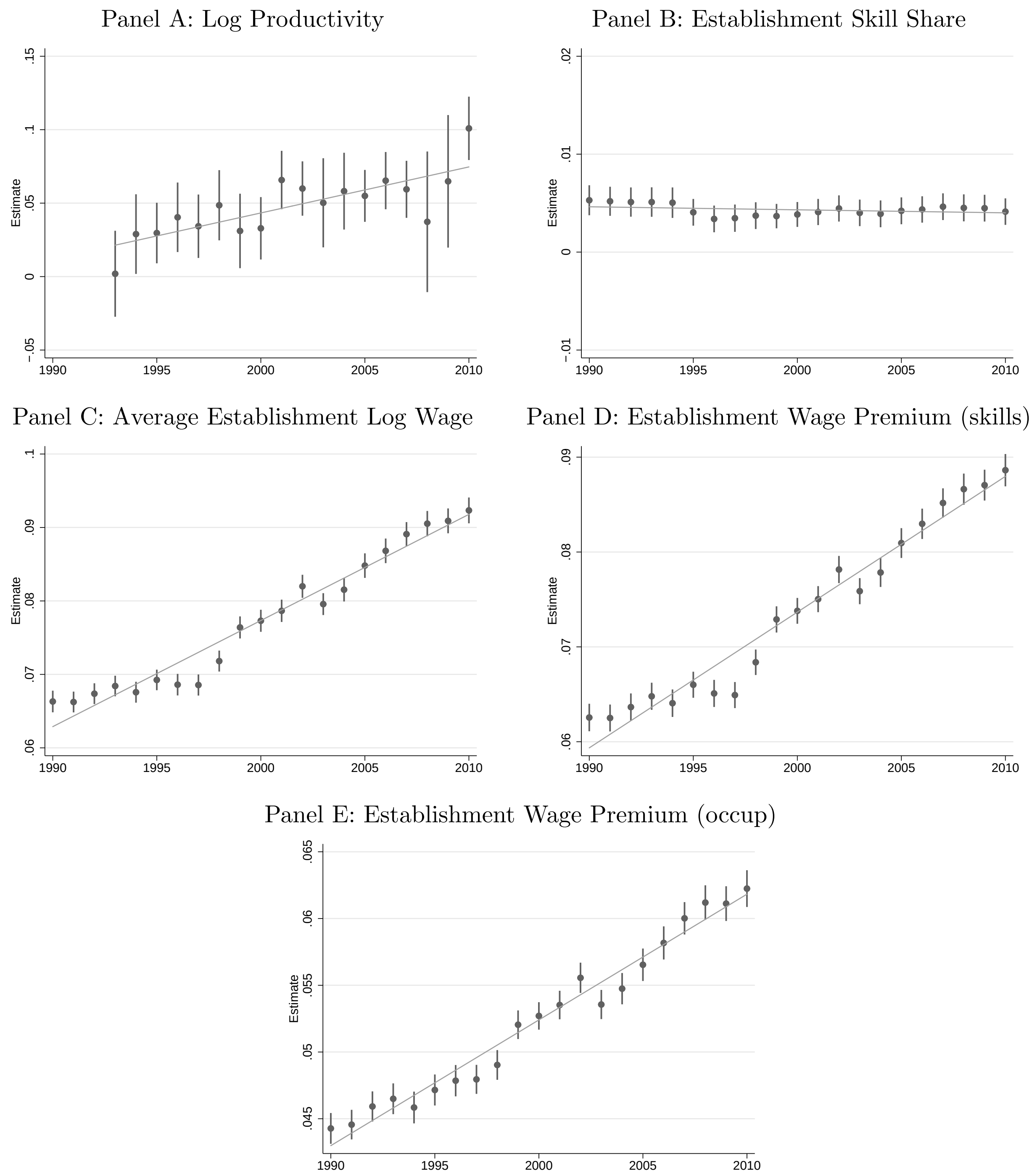
Note: The figure compares the within-industry gaps in wages for workers in different skill groups (occupations), relative to the gaps in wages for workers in the same skill group (occupation) but in different establishments. Panel A shows the evolution of the within-industry skilled wage premium and of the 90-10 and 80-20 percentile gap in establishment wage premiums (skills). Panel B displays the within-industry evolution of the 90-10 and 80-20 percentile gaps in establishment wage premiums (317 occupations) and in occupational wages. Establishment wage premiums adjust for the skill (two broad skill groups) and occupation (317 occupations) composition of the establishment, and are computed as the establishment average of the residual of an individual wage regression, estimated separately for each year, that controls for skill group (occupations) interacted with 3-digit industry fixed effects. Within-industry gaps are averaged across industries using 1990 industry shares.

Figure 4: Year-by-Year Associations between Establishment Productivity and Other Establishment Characteristics (Within Industries)



Note: The figure shows estimated coefficients and 95% confidence intervals from regressions of the outcome that appears in the title of each panel on log establishment productivity and a full set of 3-digit industry fixed effects, with the regressions estimated separately for each year. Results are based on establishments in the IABEP and observations are weighted by establishment size and survey weights. Establishment wage premiums adjust for the skill (two broad groups) and occupation (317 occupations) composition of the establishment; see text for details.

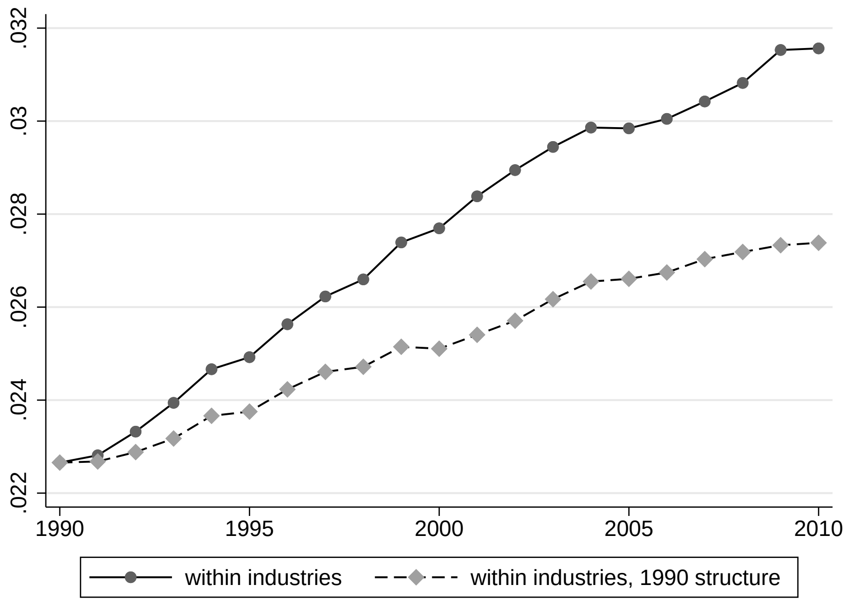
Figure 5: Year-by-Year Associations between Establishment Size and Other Establishment Characteristics (Within Industries)



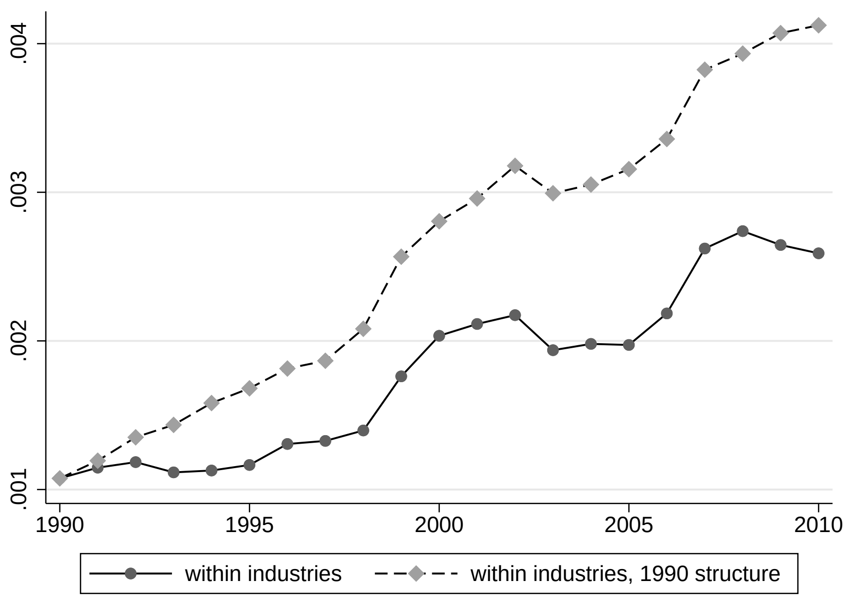
Note: The figure shows estimated coefficients and 95% confidence intervals from regressions of the outcome that appears in the title of each panel on log establishment size and a full set of 3-digit industry fixed effects, with the regressions estimated separately for each year. Results are based on establishments in the BEH and observations are weighted by establishment size (except Panel A which uses establishments in the IABEP and weights observations based on establishment size and survey weights). Establishment wage premiums adjust for the skill (two broad groups) and occupation (317 occupations) composition of the establishment; see text for details.

Figure 6: Skilled Share Heterogeneity and Sorting (Within Industries)

Panel A: Variance of Skilled Employment Shares

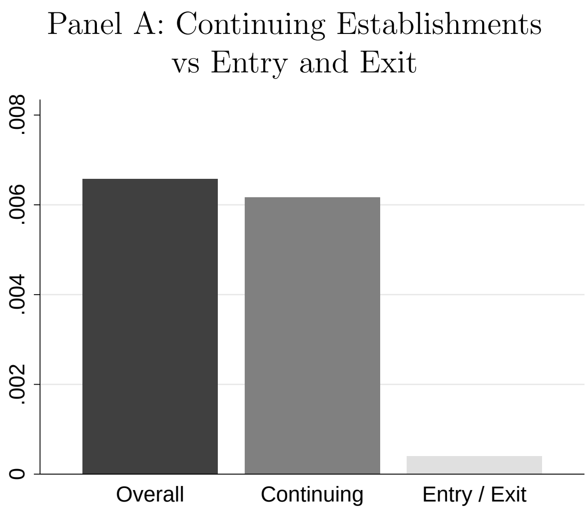


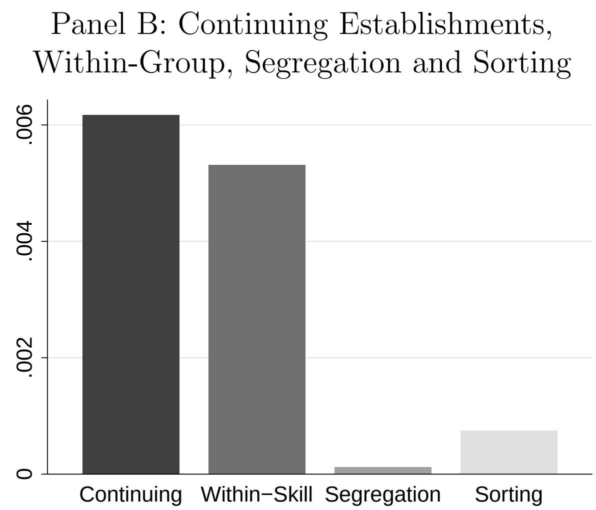
Panel B: Co-variance between Skill Share and Establishment Wage Premium

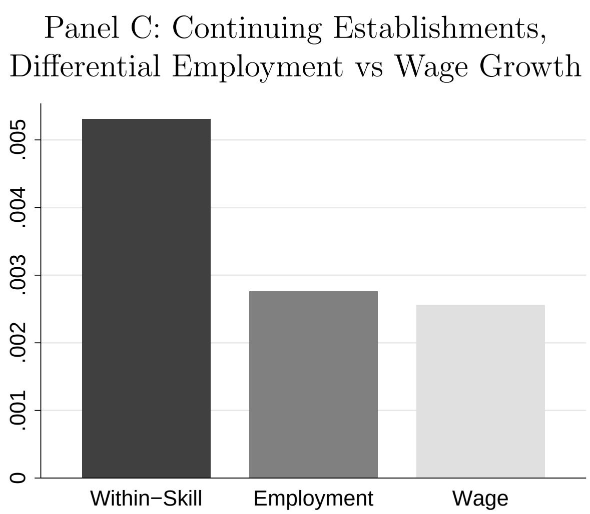


Note: Panel A shows the evolution of the variance of the skilled employment share across establishments within industries, averaging across industries using either observed industry employment shares in each year (solid line) or constant 1990 industry employment shares in all periods (dashed line); see Appendix B.6. Panel B shows the co-variance between establishments' skilled employment shares and their wage premium (skills); see Appendix B.7.

Figure 7: Decomposition of Changes in the Within-Industry Between-Establishment Variance of Log Wages



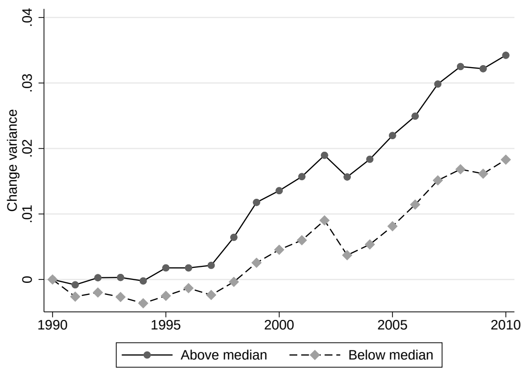




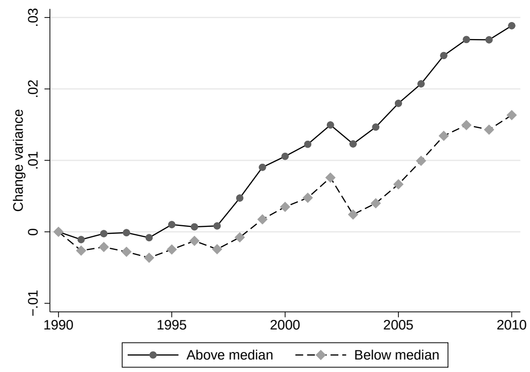
Note: Panel A decomposes changes in the within-industry, between-establishment variance of log wages into two components: selection due to establishment entry and exit, and changes among continuing establishments; see Equation (8). Panel B decomposes changes in the within-industry variance of log wages among continuing establishments into within-group, segregation and sorting components; see Equation (9). Panel C decomposes changes in the within-group component among continuing establishments into differential employment and wage growth components; see Equation (10). The decompositions are performed over non-overlapping 5-year windows within industries and then averaged across industries and time using 1990 employment shares as weights and giving equal weight to each time period.

Figure 8: Industries with Below vs Above Median Increases in Skilled Employment Shares

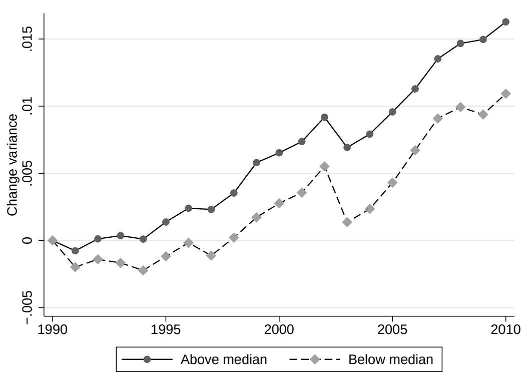
Panel A: Change in Overall Variance of Establishment (log) Wages



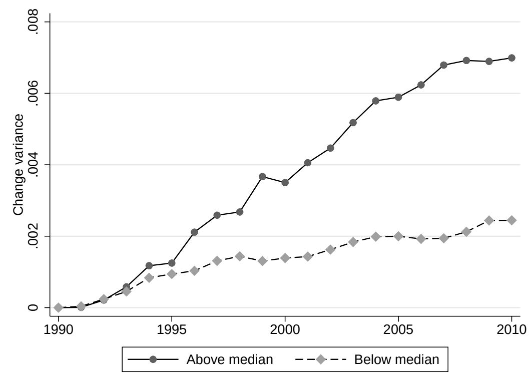
Panel B: Change in Variance of Establishment Wage Premiums (skills)



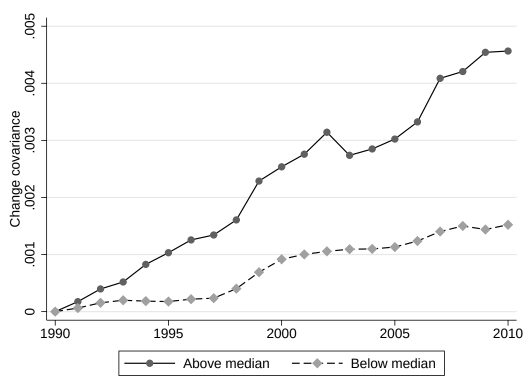
Panel C: Change in Variance of Establishment Wage Premiums (occup)



Panel D: Change in Variance of Establishment Skill Shares



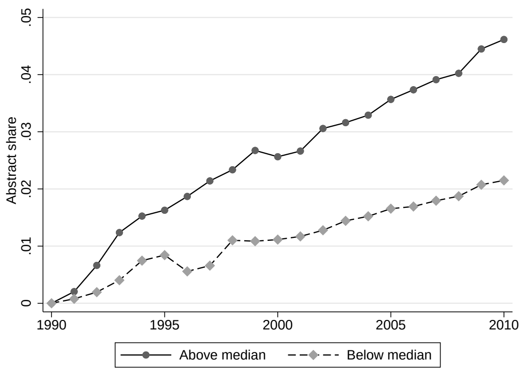
Panel E: Change in Co-variance of Establishment Wage Premiums and Skill Shares



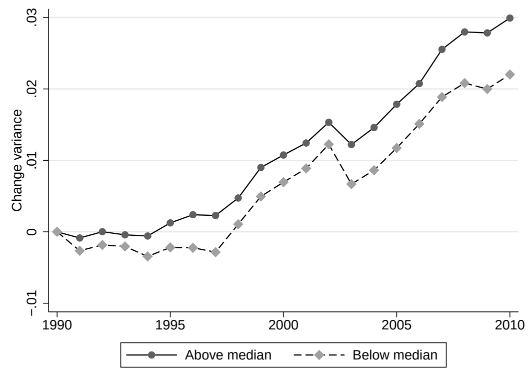
Note: The figures contrast the evolution of the overall variance of average establishment (log) wages (Panel A), the variance of establishments' wage premiums adjusting for their skill and detailed occupation structure (Panels B and C), the variance of establishments' skilled employment shares (Panel D), and the co-variance between establishments' skilled employment shares and wage premiums (Panel E) for two groups of industries: industries with below median and above median changes in the industry-level skilled employment share between 1990 and 2010 based on 196 3-digit industries. We average across industries using the 1990 industry employment structure as weights.

Figure 9: Industries with Below vs Above Median Robot Adoption

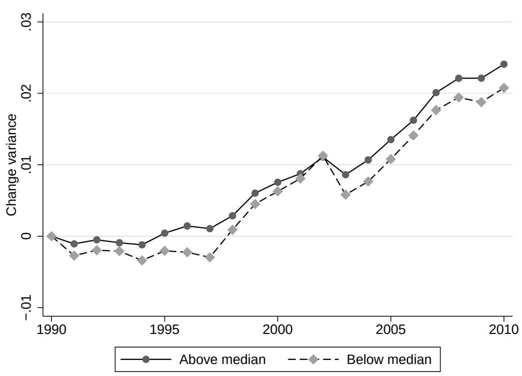
Panel A: Change in Establishment Skill Shares



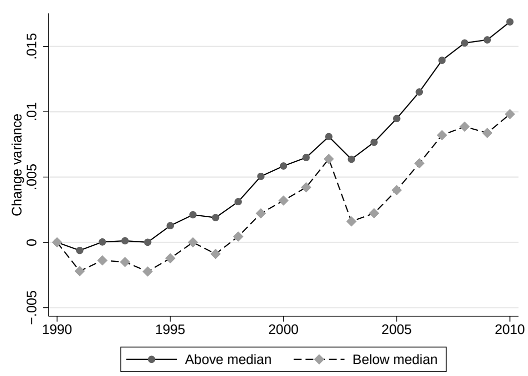
Panel B: Change in Overall Variance of Establishment (log) Wages



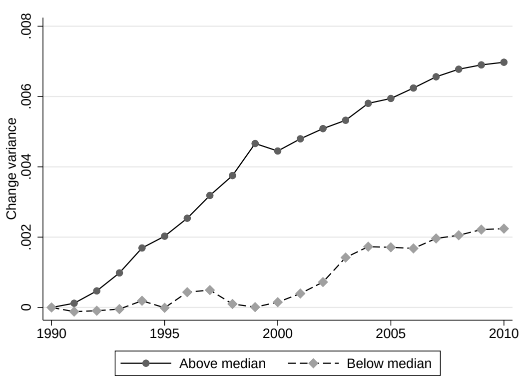
Panel C: Change in Variance of Establishment Wage Premiums (skills)



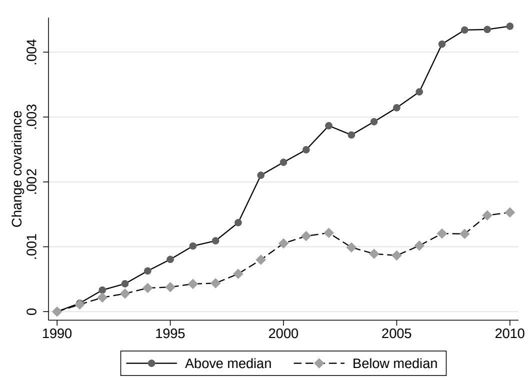
Panel D: Change in Variance of Establishment Wage Premiums (occup)



Panel E: Change in Variance of Establishment Skill Shares

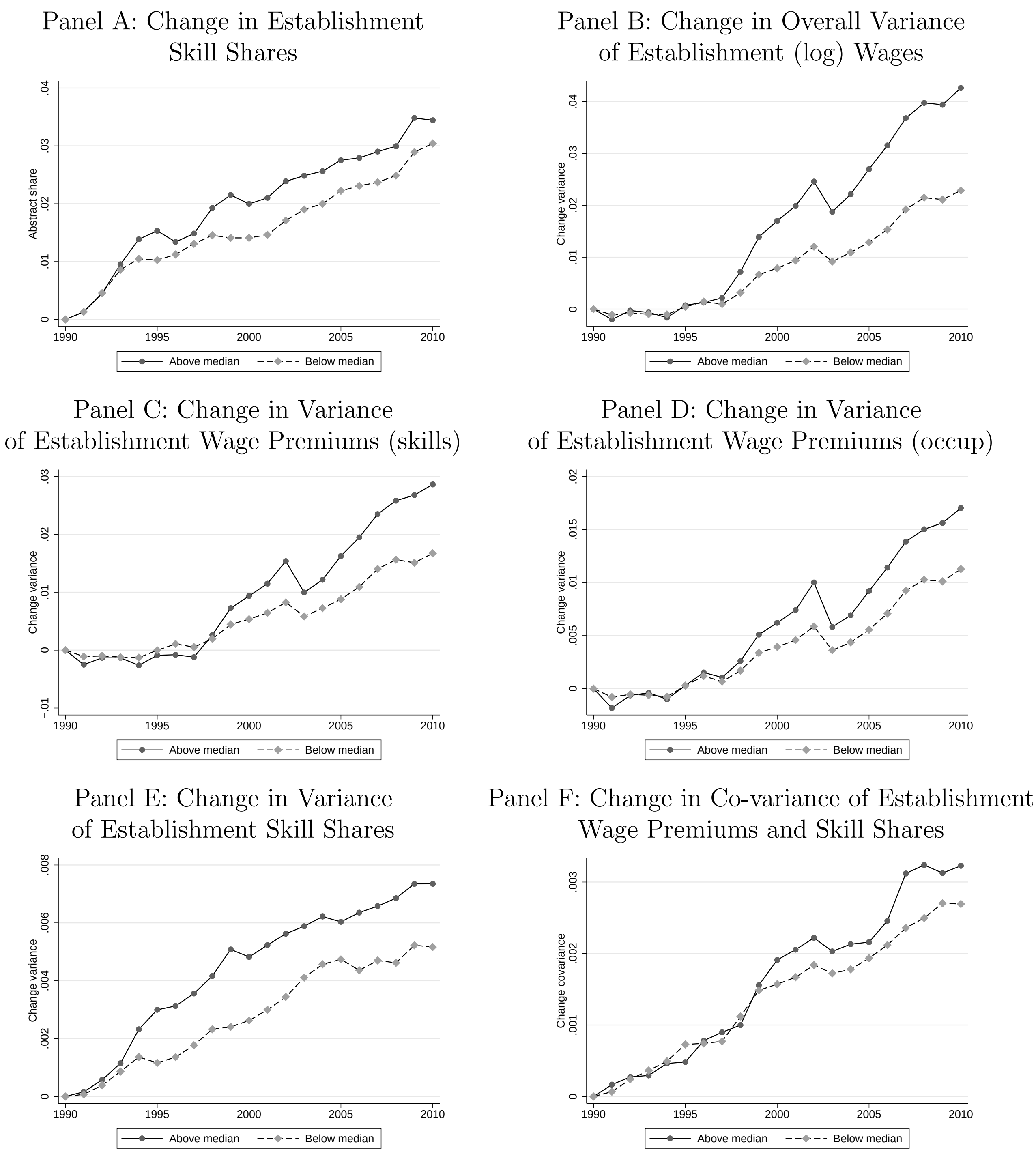


Panel F: Change in Co-variance of Establishment Wage Premiums and Skill Shares



Note: The figures contrast the evolution of the increase in the skilled employment share (Panel A), the variance of average establishment (log) wages (Panel B), the variance of establishments' wage premiums adjusting for their skill and detailed occupation structure (Panels C and D), the variance of establishments' skilled employment shares (Panel E), and the co-variance between establishments' skilled employment shares and wage premiums (Panel F) for two groups of industries: industries with below median and above median robot adoption between 1993 and 2010 based on 193 3-digit industries and data from the International Federation of Robotics. We average across industries using the 1990 industry employment structure as weights.

Figure 10: Industries with Below and Above Median ICT Capital Adoption



Note: The figures contrast the evolution of the increase in the skilled employment share (Panel A), the variance of average establishment (log) wages (Panel B), the variance of establishments' wage premiums adjusting for their skill and detailed occupation structure (Panels C and D), the variance of establishments' skilled employment shares (Panel E), and the co-variance between establishments' skilled employment shares and wage premiums (Panel F) for two groups of industries: industries with below median and above median increases in ICT capital between 1991 and 2007 based on 48 2-digit industries and EUKLEMS data. We average across industries using the 1990 industry employment structure as weights.

Table 1: Cross-Sectional Relationships between Wages, Productivity and Employment (Within Industry and Year)

Panel A: Relationship with (Log) Productivity

	Log Skilled Workers	Log Unskilled Workers	Skilled Share	Avg. Log wage	Establishment Premium (skills)	Establishment Premium (occup)	Avg. Log Wage Skilled	Avg. Log Wage Unskilled
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Productivity	0.27***	0.19***	0.0065**	0.080***	0.075***	0.056***	0.066***	0.071***
(Rev. p. Worker)	(0.031)	(0.025)	(0.0026)	(0.0050)	(0.0047)	(0.0035)	(0.0054)	(0.0055)
N	86,883	86,883	86,883	86,883	86,883	86,883	51,605	51,605

Panel B: Relationship with Establishment Size

	Productivity	Skilled Share	Avg. Log wage	Establishment Premium (skills)	Establishment Premium (occup)	Avg. Log Wage Skilled	Avg. Log Wage Unskilled
	(1)	(2)	(3)	(4)	(5)	(6)	
Log Estab Size	0.047***	0.0043***	0.077***	0.073***	0.052***	0.061***	0.060***
(# of employees)	(0.006)	(0.0006)	(0.0006)	(0.0006)	(0.0005)	(0.0009)	(0.0009)
N	86,883	26,814,744	26,814,744	26,814,744	26,814,744	5,785,470	5,785,470

Note: All regressions include a set of fully interacted 3-digit industry and year fixed effects. Productivity is measured as total sales per full-time equivalent worker using sales data from the IABEP. Establishment wage premiums adjust for the skill (two broad skill groups) and occupation (317 occupations) composition of the establishment, and are computed as the establishment average of the residual of an individual wage regression, estimated separately for each year, that controls for skill group (occupations) interacted with 3-digit industry fixed effects. For Columns (1) and (2) of Panel A establishments with no workers of a given type are imputed to have one part-time workers (i.e. 0.5 full-time equivalent workers) of that type in order to be able to compute log employment. All columns in Panel A and Column (1) in Panel B are based on establishments observed in the IABEP; observations are weighted by establishment size and survey weights. All other columns of Panel B are based on establishments observed in the full BEH data, and observations are weighted by establishment size. Standard errors are clustered at the establishment level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2: Baseline Establishment Characteristics and Within-Establishment Changes (Within Industries)

Panel A: Baseline Establishment Size and Longitudinal Changes in Other Outcomes

	Dependent Variable:				
	Δ Estab Productivity	Δ Skilled Share	Δ Avg. Log Wage	Δ Estab Premium (Skills)	Δ Estab Premium (Occup)
	(1)	(2)	(3)	(4)	(5)
Estab size at baseline	0.032***	0.0030***	0.0081***	0.0069***	0.0047***
	(0.0096)	(0.00025)	(0.00034)	(0.00028)	(0.00023)
N	5,460	3,452,385	3,452,385	3,452,385	3,452,385

Panel B: Baseline Establishment Characteristics and Longitudinal Changes in Size

	Dependent Variable: Δ Employment (Incl. Exits)				
	Independent Variable: Baseline Level of ...				
	Estab Productivity	Skilled Share	Avg. Log Wage	Estab Premium (Skills)	Estab Premium (Occup)
	(1)	(2)	(3)	(4)	(5)
$\hat{\beta}$	0.049***	0.037***	0.11***	0.12***	0.14***
	(0.0084)	(0.0079)	(0.0042)	(0.0041)	(0.0039)
N	15,782	5,107,149	5,107,149	5,107,149	5,107,149

Note: Panel A shows estimated coefficients from regressions of within-establishment changes in the outcome variable shown in each column of the table on baseline establishment size (conditioning on surviving establishments). Panel B shows estimated coefficients from regressions of within-establishment employment growth (including exiting establishment) on the baseline establishment characteristic shown in each column. Within-establishment changes are taken over non-overlapping 5-year windows. Regressions include a set of fully interacted 3-digit industry and year fixed effects. With the exception of Column (1), results are based on establishments in the full BEH and observations are weighted by establishment size. Column (1) uses establishments in the IABEP. Productivity is measured as total sales per full-time equivalent worker using sales data from the IABEP, and observations are weighted based on total employment and survey weights. Establishment wage premiums adjust for the skill (two broad groups) and occupation (317 occupations) composition of the establishment; see text for details. Standard errors are clustered at the establishment level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Online Appendix for:

Technological Change, Firm Heterogeneity and Wage Inequality

Guido Matias Cortes (York University)
Adrian Lerche (Institute for Employment Research)
Uta Schönberg (University College London and Institute for Employment Research)
Jeanne Tschopp (University of Bern)

Appendix A Data

Appendix A.1 Imputation of Censored Wages

To impute top-coded wages, we first define age-education cells based on five age groups (with 10-year intervals) and three education groups (no post-secondary education, vocational degree, college or university degree). Within each of these cells, following Dustmann et al. (2009) and Card et al. (2013), we estimate Tobit wage equations separately by year while controlling for age; firm size (quadratic, and a dummy for firm size greater than 10); occupation dummies; the focal worker's mean wage and mean censoring indicator (each computed over time but excluding observations from the current time period); and the firm's mean wage, mean censoring indicator, mean years of schooling, and mean university degree indicator (each computed at the current time period by excluding the focal worker observations). For workers observed in only one time period, the mean wage and mean censoring indicator are set to sample means, and a dummy variable is included. A wage observation censored at value c is then imputed by the value $\mathrm{X}\hat{\beta} + \hat{\sigma}\Phi^{-1}[\mathrm{k} + \mathrm{u}(1 - \mathrm{k})]$, where Φ is the standard normal CDF, u is drawn from a uniform distribution, $\mathrm{k} = \Phi[(\mathrm{c} - \mathrm{X}\hat{\beta})/\hat{\sigma})]$ and $\hat{\beta}$ and $\hat{\sigma}$ are estimates for the coefficients and standard deviation of the error term from the Tobit regression.

Appendix A.2 Harmonization of Industry Codes

In 1999, the industry classification in the BEH social security data changed in order to make the industry classification compatible with international NAICS codes. In 1999, both the old and new industry code are included for all establishments. We use this information to compute the most common NAICS industry code for each old industry code. For establishments that still exist in 1999, we assign their 1999 NAICS industry code for all earlier years.

For establishments that have exited by 1999, we assign the NAICS industry code that, in 1999, is the most common given the establishment's old industry code.

Appendix B Additional Details on the Empirical Analysis

Appendix B.1 Employment Share of Skilled Workers (Figure 1, Panel A)

The actual employment share of skilled workers in year t (the black line in Figure 1, Panel A), S_t, is a weighted average of S_{kt}, the employment share of skilled workers in each industry k:

$$S_t = \sum_k \frac{n_{kt}}{n_t} S_{kt}, \tag{B.1}$$

where n_{kt} and n_t denote the number of workers employed in industry k at time t and the total number of employed workers at time t, respectively. Hence, (n_{kt}/n_t) denotes industry k's share of employment in year t.

The counterfactual employment share of skilled workers in year t holding the industry structure constant at its 1990 employment level (the grey line in Figure 1, Panel A) can then be computed as:

$$S_t^{1990} = \sum_k \frac{n_{k1990}}{n_{1990}} S_{kt}, \tag{B.2}$$

where (n_{k1990}/n_{1990}) captures industry k's share of employment in 1990.

Appendix B.2 Skilled Wage Premium (Figure 1, Panel B)

The skilled wage premium, $SkillPrem_t$, at time t (the black line in Figure 1, Panel B) is computed as follows:

$$SkillPrem_t = \sum_k \frac{n_{kt}}{n_t} \underbrace{\left(\overline{\ln w}_{kt}^{\,s} - \overline{\ln w}_{kt}^{\,r}\right)}_{SkillPrem_{kt}}, \tag{B.3}$$

where $\overline{\ln w}_{kt}^{\,s}$ and $\overline{\ln w}_{kt}^{\,r}$ denote the average log wage of skilled and unskilled workers in industry k at time t, respectively. Thus, the skilled wage premium is a weighted average of the difference between the average log wage of skilled and unskilled workers by industry and year, averaged across industries using current industrial employment shares as weights. The term $SkillPrem_{kt}$ denotes the industry-year-specific skilled wage premium.

The counterfactual skilled wage premium in year t holding the industry structure constant at its 1990 employment level (the grey line in Figure 1, Panel B) is computed as:

$$SkillPrem_t^{1990} = \sum_k \frac{n_{k1990}}{n_{1990}} \left(\overline{\ln w}_{kt}^{s} - \overline{\ln w}_{kt}^{r} \right). \tag{B.4}$$

Appendix B.3 Wage Inequality Within Skill Groups (Figure 1, Panel C)

We compute wage inequality within skill groups in year t, denoted Var_t^{WT}, as follows:

$$\begin{aligned} Var_t^{WT} &= \sum_\ell \frac{n_{\ell t}}{n_t} \left[\frac{1}{n_{\ell t}} \sum_{i \in i_{\ell t}} (\ln w_{it} - \overline{\ln w_{\ell t}})^2 \right] \\ &= \frac{1}{n_t} \sum_\ell \sum_{i \in i_{\ell t}} (\ln w_{it} - \overline{\ln w_{\ell t}})^2, \end{aligned} \tag{B.5}$$

where ℓ represents a skill, i denotes an individual, $i_{\ell t}$ is the set of individuals in skill ℓ at time t, $n_{\ell t}$ is the total number of workers in skill ℓ at time t, $\ln w_{it}$ is the log wage of individual i at time t and $\overline{\ln w_{\ell t}}$ is the average log wage in skill ℓ at time t. Thus, within-group wage inequality is a weighted average of the variance of individual wages by skill, averaged over skills using the actual share of employment in each skill. Within-occupation wage inequality is computed similarly, with ℓ denoting 3-digit occupations instead of the two broad skill categories.

Appendix B.4 Within-Industry Between-Establishment Wage Inequality (Figure 2, Panel B)

We compute the counterfactual within-industry between-establishment wage inequality using the 1990 industry structure, denoted by $Var_t^{WIBE,1990}$, as follows:

$$Var_t^{WIBE,1990} = \sum_k \frac{n_{k1990}}{n_{1990}} \underbrace{\sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (\overline{\ln w}_{ft} - \overline{\ln w}_{kt})^2}_{Var_{kt}(\overline{\ln w}_{ft})}. \tag{B.6}$$

where f indexes establishments, f_{kt} is the set of establishments in industry k in year t, n_{ft} is the total number of workers at establishment f in year t, $\overline{\ln w}_{ft}$ is the average log wage in establishment f at time t and $\overline{\ln w_{kt}}$ is the average log wage in industry k at time t. The term $Var_{kt}(\overline{\ln w_{ft}})$ refers to the variance of establishment log wages in industry k at time t.

Appendix B.5 Within-Industry Between-Establishment Wage Premium Inequality (Figure 2, Panel C)

To compute the establishment wage premiums (two skills), which we denote as $\widetilde{\ln w_{ft}}$, we first run a regression, separately for each year, of individual log wages on an indicator variable that is equal to 1 if the individual is a skilled worker (i.e. working in a professional, managerial or technical occupation) in year t, interacted with K indicator variables that are equal to 1 if the individual is employed in industry k in that year:

$$\ln w_{i(k)t} = \sum_k \beta_{kt} A_{i(k)t} \times D_{i(k)t} + \epsilon_{i(k)t}, \tag{B.7}$$

where $\ln w_{i(k)t}$ is the log wage of individual i employed in industry k at time t, $A_{i(k)t}$ denotes the skill indicator, $D_{i(k)t}$ is the industry indicator and $\epsilon_{i(k)t}$ is the error term. The establishment wage premium (skills) of establishment f in year t is then computed as the residuals from the estimation of equation (B.7), averaged across individuals in the establishment:

$$\widetilde{\ln w_{ft}} = \frac{1}{n_{ft}} \sum_{i \in i_{ft}} \widehat{\epsilon}_{i(k)t}, \tag{B.8}$$

where i_{ft} is the set of individuals working in establishment f in year t.

We proceed similarly to compute the establishment wage premiums (occupations), replacing the indicator variable $A_{i(k)t}$ in Equation (B.7) with a full set of occupation fixed effects, thus allowing for occupation wage premiums to differ across industries in each year.

We compute the counterfactual variance of within-industry between-establishment wage premiums using the 1990 industry structure, denoted $Var_t^{\widetilde{WIBE},1990}$, as follows:

$$Var_t^{\widetilde{WIBE},1990} = \sum_k \frac{n_{k1990}}{n_{1990}} \sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (\widetilde{\ln w_{ft}} - \overline{\widetilde{\ln w_{kt}}})^2,$$

where $\widetilde{\ln w_{ft}}$ denotes the wage premium of establishment f at time t, and $\overline{\widetilde{\ln w_{kt}}}$ is the average establishment wage premium in industry k at time t.

Appendix B.6 Within-Industry Heterogeneity in the Employment Share of Skilled Workers (Figure 6, Panel A)

Within-industry heterogeneity in establishments' employment share of skilled workers, denoted Var_t^A, is given by the within-industry variance in establishments' employment share of skilled workers, averaged over industries using industrial employment shares as weights:

$$Var_t^A = \sum_k \frac{n_{kt}}{n_t} \sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (S_{ft} - S_{kt})^2,$$

where S_{ft} is establishment f's employment share of skilled workers at time t.

The counterfactual within-industry variance in establishments' employment shares of skilled workers, holding the industry structure constant at its 1990 level, $Var_t^{A,1990}$, equals:

$$Var_t^{A,1990} = \sum_k \frac{n_{k1990}}{n_{1990}} \sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (S_{ft} - S_{kt})^2.$$

Appendix B.7 Within-Industry Sorting (Figure 6, Panel B)

We capture the extent of sorting of skilled workers into high-wage establishments using the within-industry co-variance between establishments' employment shares of skilled workers and their wage premiums, averaged across industries using industrial employment shares. This co-variance, denoted Cov_t, is computed as follows:

$$Cov_t = \sum_k \frac{n_{kt}}{n_t} \sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (S_{ft} - S_{kt})(\widetilde{\ln w_{ft}} - \overline{\widetilde{\ln w_{kt}}}),$$

and the corresponding counterfactual co-variance, holding the industry structure constant at its 1990 level, denoted Cov_t^{1990} , equals:

$$Cov_t^{1990} = \sum_k \frac{n_{k1990}}{n_{1990}} \sum_{f \in f_{kt}} \frac{n_{ft}}{n_{kt}} (S_{ft} - S_{kt})(\widetilde{\ln w_{ft}} - \overline{\widetilde{\ln w_{kt}}}).$$

Appendix C Model

This section contains details of the model and of derivations that were omitted in the main text. The presentation is not necessarily self-contained but rather complementary with Section 4 of the paper. We also refer the reader to Section 5.4 of the technical appendix of Helpman et al. (2010) for more details on the model with a CES production function and two types of workers.

For the derivations below, it is useful to note that $\varphi(\theta)$ and $\phi_\ell(\theta)$, where $\ell \in \{s, r\}$, are defined as follows:

$$\varphi(\theta) \equiv \frac{\mu_s^\nu \left(\theta \overline{a}_s h_s^\gamma\right)^\nu}{\left(\overline{a}_r h_r^\gamma\right)^\nu}, \qquad \phi_s(\theta) \equiv \frac{\varphi(\theta)}{1+\varphi(\theta)}, \qquad \phi_r(\theta) \equiv \frac{1}{1+\varphi(\theta)}. \tag{C.1}$$

Appendix C.1 Derivations of the Key Equilibrium Relationships

This section derives the equilibrium relationships for the variables which play a crucial role when examining the impact of skill-biased technological change on wage inequality.

Appendix C.1.1 Firm-level Equilibrium Variables

Below we use the following first-order conditions from the profit maximization problem to derive firm-level equilibrium revenue, employment and wages by skill:

$$\frac{\beta\gamma}{1+\beta\gamma}\phi_\ell(\theta) r(\theta) = b_\ell n_\ell(\theta) \tag{C.2}$$

$$\frac{\beta(1-\gamma k)}{1+\beta\gamma}\phi_\ell(\theta) r(\theta) = c\widetilde{a}_\ell(\theta)^\delta \tag{C.3}$$

Revenue As Helpman et al. (2010) mention in Appendix 5.4 footnote 1, revenue can be expressed as:

$$r(\theta) = \kappa_y^\beta A \left[1+\varphi(\theta)\right]^{\beta/\nu} \left[\widetilde{a}_r(\theta)^{1-k\gamma} n_r(\theta)^\gamma\right]^\beta, \tag{C.4}$$

where $\kappa_y \equiv \frac{k a_{min}^{\gamma k}}{k-1}$. Using the first-order conditions along with equation (C.4) and the definition of ϕ_r, one obtains the revenue equation:

$$r(\theta) = \kappa_r \left[1+\varphi(\theta)\right]^{\frac{\beta\Lambda}{\nu\Gamma}}, \tag{C.5}$$

where κ_r is equivalent to:

$$\kappa_r \equiv A^{1/\Gamma} \left[\kappa_y \left(\frac{\beta}{1+\beta\gamma}\right)^{\frac{1-k\gamma}{\delta}+\gamma} \left(\frac{1-\gamma k}{c}\right)^{\frac{1-k\gamma}{\delta}} \left(\frac{\gamma}{b_r}\right)^\gamma\right]^{\beta/\Gamma}. \tag{C.6}$$

Employment by skill and the employment share of skilled workers To obtain firm-level employment, note that from equation (C.2):

$$\begin{aligned} n_r(\theta) &= \frac{\beta\gamma}{1+\beta\gamma}\left[1+\varphi(\theta)\right]^{-1} b_r^{-1} r(\theta) \\ &= \left(\frac{\beta\gamma}{1+\beta\gamma}\right) b_r^{-1}\kappa_r \left[1+\varphi(\theta)\right]^{\frac{\beta-\nu}{\nu\Gamma}}, \end{aligned} \tag{C.7}$$

where $\frac{\beta-\nu}{\nu\Gamma} = \frac{\beta\Lambda}{\nu\Gamma} - 1 > 0$, and from equation (C.3):

$$\begin{aligned} \widetilde{a}_r(\theta) &= \left\{\frac{\beta(1-\gamma k)}{1+\beta\gamma}\left[1+\varphi(\theta)\right]^{-1} c^{-1} r(\theta)\right\}^{1/\delta} \\ &= \left[\frac{\beta(1-\gamma k)}{1+\beta\gamma}\right]^{1/\delta} c^{-1/\delta}\kappa_r^{1/\delta}\left[1+\varphi(\theta)\right]^{\frac{\beta-\nu}{\delta\nu\Gamma}}. \end{aligned} \tag{C.8}$$

Using expression $h_\ell(\theta) = n_\ell(\theta)\left(\frac{a_{min}}{\widetilde{a}_\ell(\theta)}\right)^k$, along with (C.7) and (C.8), we have that:

$$h_r(\theta) = n_r(\theta)\left(\frac{a_{min}}{\widetilde{a}_r(\theta)}\right)^k$$

$$= \left(\frac{\beta\kappa_r}{1+\beta\gamma}\right)^{1-k/\delta}\left(\frac{c}{1-\gamma k}\right)^{k/\delta} b_r^{-1} a_{min}^k \left[1+\varphi(\theta)\right]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)} \tag{C.9}$$

$$= h_{dr}\left[1+\varphi(\theta)\right]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)}, \tag{C.10}$$

where:

$$h_{dr} \equiv \left(\frac{\beta\kappa_r}{1+\beta\gamma}\right)^{1-k/\delta}\left(\frac{c}{1-\gamma k}\right)^{k/\delta} b_r^{-1} a_{min}^k. \tag{C.11}$$

Proceeding in a similar way for firm-level employment of skilled workers, we obtain:

$$h_s(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-k/\delta} h_r(\theta), \tag{C.12}$$

and it follows that the firm's employment share of skilled workers is given by:

$$\frac{h_s(\theta)}{h(\theta)} = \frac{b_r\varphi(\theta)^{1-k/\delta}}{b_s + b_r\varphi(\theta)^{1-k/\delta}}, \tag{C.13}$$

where $h(\theta) = h_s(\theta) + h_r(\theta)$.

Wages by skill To derive equilibrium firm-level wages by skill, it is useful to note that the solution of the Stole and Zwiebel bargaining game takes the following form:

$$w_\ell(\theta) = \frac{\beta\gamma}{1+\beta\gamma}\frac{\phi_\ell(\theta)r(\theta)}{h_\ell(\theta)} \tag{C.14}$$

Using (C.14) along with (C.4) and (C.10), we have that firm wages of unskilled workers are given by:

$$\begin{aligned} w_r(\theta) &= \frac{\beta\gamma}{1+\beta\gamma}\phi_r(\theta)\frac{r(\theta)}{h_r(\theta)} \\ &= \left(\frac{\beta\gamma}{1+\beta\gamma}\right)\left(\frac{\kappa_r}{h_{dr}}\right)[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}} \qquad \text{(C.15)} \\ &= w_{dr}\,[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}}, \qquad \text{(C.16)} \end{aligned}$$

where:

$$w_{dr} \equiv \left(\frac{\beta\gamma}{1+\beta\gamma}\right)\left(\frac{\kappa_r}{h_{dr}}\right). \tag{C.17}$$

Proceeding in a similar way for firm-level wages of skilled workers, we obtain:

$$w_s(\theta) = \frac{b_s}{b_r}\varphi(\theta)^{k/\delta}w_r(\theta). \tag{C.18}$$

Finally, combining the definition of $\varphi(\theta)$ together with the first-order conditions of the profit maximization problem, we obtain:

$$\varphi(\theta) = \mu_s^{\nu/\Lambda}\left(\frac{b_s}{b_r}\right)^{-\gamma\nu/\Lambda}\theta^{\nu/\Lambda}. \tag{C.19}$$

Hence, $\varphi(\theta)$ only depends on firm productivity and parameters.

Revenue per worker Combining equation (C.5) together with equations (C.10) and (C.12), we can express revenue per worker as a function of productivity (through $\varphi(\theta)$) and parameters only:

$$\begin{aligned} \frac{r(\theta)}{h(\theta)} &= \kappa_r\,[1+\varphi(\theta)]^{\frac{\beta\Lambda}{\nu\Gamma}}\left[\frac{b_r\varphi(\theta)^{1-k/\delta}+b_s}{b_s}\right]^{-1}[h_r(\theta)]^{-1} \\ &= \left(\frac{\kappa_r}{h_{dr}}\right)\left[\frac{b_s}{b_r\varphi(\theta)^{1-k/\delta}+b_s}\right][1+\varphi(\theta)]^{1+\frac{k}{\delta}\left(\frac{\beta-\nu}{\nu\Gamma}\right)} \qquad \text{(C.20)} \end{aligned}$$

Appendix C.1.2 Determination of the Productivity Threshold

As is standard in Melitz-type heterogeneous firm models, the productivity threshold for production, θ_d, is pinned down by both the Zero-Cutoff Profit (ZCP) and the Free Entry (FE) conditions.

The ZCP condition, which requires that the firm at the cutoff θ_d makes zero profits, implies:[30]

$$\frac{\Gamma}{1+\beta\gamma} r(\theta_d) = f_d. \tag{C.21}$$

Moreover, given equation (C.5), relative revenues across two firms with productivities θ_1 and θ_2 can be written as:

$$\frac{r(\theta_1)}{r(\theta_2)} = \left[\frac{1+\varphi(\theta_1)}{1+\varphi(\theta_2)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}}. \tag{C.22}$$

Combining equation (C.22) along with the ZCP condition (C.21) we obtain:

$$r(\theta) = f_d \left(\frac{\Gamma}{1+\beta\gamma}\right)^{-1} \left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}}. \tag{C.23}$$

The FE condition states that the expected profits for a potential entrant should equal the fixed entry cost:

$$\int_{\theta_d}^{\infty} \pi(\theta) dG(\theta) = f_e. \tag{C.24}$$

Therefore, combining equations (C.23) and (C.24) implies:

$$f_d \int_{\theta_d}^{\infty} \left(\left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)}\right]^{\frac{\beta\Lambda}{\nu\Gamma}} - 1 \right) dG(\theta) = f_e. \tag{C.25}$$

Equation (C.25) pins down the equilibrium threshold θ_d as a function of the parameters of the model and the search costs b_s and b_r.

Appendix C.2 Relationship between Firm-Specific Equilibrium Outcomes and θ

This section presents the proofs for the results in Equations (4) and (7), and shows that revenue per worker is monotonically increasing in θ.

[30] This is obtained by noting that profits can be written as:

$$\pi(\theta) = \frac{\Gamma}{1+\beta\gamma} r(\theta) - f_d.$$

First, note that:

$$\frac{\partial \varphi(\theta)}{\partial \theta} = \frac{\nu}{\Lambda}\mu_s^{\frac{\nu}{\Lambda}}\left(\frac{b_s}{b_r}\right)^{-\frac{\gamma\nu}{\Lambda}}\theta^{\frac{\nu}{\Lambda}-1} \quad > \quad 0, \tag{C.26}$$

and recall that $\beta > \nu$, $\Lambda > \Gamma$ and $\delta > k$ such that $\frac{\beta-\nu}{\nu\Gamma} > 0$ and $1 - \frac{k}{\delta} > 0$.

Proof of Equation (4): Taking the derivative of equations (C.10) and (C.12), we obtain:

$$\begin{aligned}
\frac{\partial h_r(\theta)}{\partial \theta} &= h_{dr}\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)-1}\cdot\frac{\partial \varphi(\theta)}{\partial \theta} && > \quad 0\\
\frac{\partial h_s(\theta)}{\partial \theta} &= \frac{b_r}{b_s}\left[\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\cdot\frac{\partial \varphi(\theta)}{\partial \theta}\cdot h_r(\theta)+\varphi(\theta)^{1-\frac{k}{\delta}}\cdot\frac{\partial h_r(\theta)}{\partial \theta}\right] && > \quad 0
\end{aligned} \tag{C.27}$$

Taking the derivative of equation (C.13), we have that:

$$\frac{\partial}{\partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{b_s b_r\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\cdot\frac{\partial \varphi(\theta)}{\partial \theta}}{\left[b_s+b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]^2} \quad > \quad 0 \tag{C.28}$$

Proof of Equation (7): Taking the derivative of equations (C.16) and (C.18), and of the ratio of the two, we obtain:

$$\begin{aligned}
\frac{\partial w_r(\theta)}{\partial \theta} &= w_{dr}\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}[1+\varphi(\theta)]^{\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}-1}\cdot\frac{\partial \varphi(\theta)}{\partial \theta} && > \quad 0\\
\frac{\partial w_s(\theta)}{\partial \theta} &= \frac{b_s}{b_r}\left[\frac{k}{\delta}\varphi(\theta)^{\frac{k}{\delta}-1}\cdot\frac{\partial \varphi(\theta)}{\partial \theta}\cdot w_r(\theta)+\varphi(\theta)^{\frac{k}{\delta}}\cdot\frac{\partial w_r(\theta)}{\partial \theta}\right] && > \quad 0\\
\frac{\partial}{\partial \theta}\left[\frac{w_s(\theta)}{w_r(\theta)}\right] &= \frac{b_s}{b_r}\frac{k}{\delta}\varphi(\theta)^{\frac{k}{\delta}-1}\cdot\frac{\partial \varphi(\theta)}{\partial \theta} && > \quad 0
\end{aligned} \tag{C.29}$$

Proof of Relationship between Labor Productivity (Revenue per Worker) and θ: Taking the derivative of equation (C.20), we obtain:

$$\begin{aligned}
\frac{\partial}{\partial \theta}\left[\frac{r(\theta)}{h(\theta)}\right] = \left(\frac{\kappa_r}{h_{dr}}\right) b_s \frac{[1+\varphi(\theta)]^{\frac{k}{\delta}\left(\frac{\beta-\nu}{\nu\Gamma}\right)}}{\left[b_r\varphi(\theta)^{1-k/\delta}+b_s\right]^2}\frac{\partial \varphi(\theta)}{\partial \theta}\cdot\Big\{\frac{k}{\delta}\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left[b_r\varphi(\theta)^{1-k/\delta}+b_s\right]\\
+ b_r\frac{k}{\delta}\varphi(\theta)^{1-k/\delta}+\left[b_s-b_r(1-\frac{k}{\delta})\varphi(\theta)^{-k/\delta}\right]\Big\}
\end{aligned}$$

Since we assume that $\frac{b_s}{b_r}\varphi(\theta)^{k/\delta} > 1$ (which is in line with the empirical evidence that firms pay a skill wage premium; i.e. $\frac{w_s(\theta)}{w_r(\theta)} > 1$) and recalling that $(1 - \frac{k}{\delta}) \in (0, 1)$, we have that $\left[b_s - b_r(1 - \frac{k}{\delta})\varphi(\theta)^{-k/\delta}\right] > 0$. It follows that:

$$\frac{\partial}{\partial\theta}\left[\frac{r(\theta)}{h(\theta)}\right] > 0 \tag{C.30}$$

Hence, although both revenues and employment are increasing in θ, revenue per worker is monotonically increasing in θ.

Appendix C.3 Impact of Skill-Biased Technological Change

In order to evaluate how skill-biased technological change (SBTC) – modelled as an increase in the parameter μ_s – affects firms differentially across the productivity distribution, we examine the second-order derivative of firm outcome variables, with respect to both the common skill-augmenting technology parameter μ_s and firm productivity. To this end note that:

$$\frac{\partial\varphi(\theta)}{\partial\mu_s} = \frac{\nu}{\Lambda}\mu_s^{-1}\varphi(\theta) \;>\; 0 \;, \qquad \frac{\partial^2\varphi(\theta)}{\partial\mu_s\partial\theta} = \left(\frac{\nu}{\Lambda}\right)^2 \mu_s^{-1}\theta^{-1}\varphi(\theta) \;>\; 0 \;, \tag{C.31}$$

and

$$\frac{\partial\varphi(\theta)}{\partial\mu_s}\cdot\frac{\partial\varphi(\theta)}{\partial\theta} = \varphi(\theta)\cdot\frac{\partial^2\varphi(\theta)}{\partial\mu_s\partial\theta} > 0 \;. \tag{C.32}$$

Prediction 1: *Increased Skilled Wage Premium* – SBTC increases the skilled wage premium within all firms, and in the aggregate.

Proof: Taking the first-order derivative of (C.16), we have:

$$\frac{\partial}{\partial\mu_s}\left[\frac{w_s(\theta)}{w_r(\theta)}\right] \;=\; \frac{k\nu}{\delta\Lambda}\mu_s^{-1}\frac{w_s(\theta)}{w_r(\theta)} \;>\; 0 \tag{C.33}$$

Prediction 2: *Selection* – SBTC increases the productivity threshold for production θ_d.

Proof: We prove Prediction 2 by contradiction. Consider equation (C.25), which pins down the equilibrium threshold as a function of parameters of the model:

$$f_d \int_{\theta_d}^{\infty} \left(\left[\frac{1+\varphi(\theta)}{1+\varphi(\theta_d)} \right]^{\frac{\beta\Lambda}{\nu\Gamma}} - 1 \right) dG(\theta) = f_e \tag{C.34}$$

Suppose first that SBTC has no effect on θ_d. Holding θ_d fixed, the increase in $[1+\varphi(\theta)]/[1+\varphi(\theta_d)]$ induced by the increase in μ_s would imply an increase in the term in the square brackets for all relevant values of θ evaluated in the integral. Hence, with a fixed θ_d the LHS of equation (C.34) would increase while the RHS would remain fixed. This implies that θ_d cannot remain constant if μ_s increases.

Suppose now that θ_d falls as a reaction to the increase in μ_s. This would lead to a further increase in the value of the term in the square brackets for all relevant values of θ (as there would now be a larger gap between θ and θ_d). At the same time, a fall of θ_d would increase the range of values of θ that are integrated over. Hence, a decrease in θ_d would unambiguously increase the LHS of equation (C.34) while the RHS would remain fixed. This implies that θ_d cannot decrease either.

This proves that the only change in θ_d consistent with condition (C.34) is an increase in θ_d when μ_s increases. Therefore:

$$\frac{\partial \theta_d}{\partial \mu_s} \quad > \quad 0 \tag{C.35}$$

Prediction 3: *Differential Employment Growth* – SBTC strengthens the cross-sectional association between employment and productivity.

Proof: Taking the first- and second-order derivatives of (C.10) and (C.12), we obtain:

$$\frac{\partial h_r(\theta)}{\partial \mu_s} = \left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right) h_r(\theta)\left[1+\varphi(\theta)\right]^{-1} \frac{\nu}{\Lambda}\mu_s^{-1}\varphi(\theta)$$

$$\frac{\partial^2 h_r(\theta)}{\partial \mu_s \partial \theta} = \left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right) h_r(\theta)\left[1+\varphi(\theta)\right]^{-2} \cdot \frac{\partial^2 \varphi(\theta)}{\partial \mu_s \partial \theta}\left[1+\varphi(\theta)\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)\right] \quad > \quad 0$$

$$\frac{\partial h_s(\theta)}{\partial \mu_s} = \frac{\nu}{\Lambda}\mu_s^{-1}\left(1-\frac{k}{\delta}\right)\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\cdot\frac{\varphi(\theta)}{1+\varphi(\theta)}\right] h_s(\theta)$$

$$\frac{\partial^2 h_s(\theta)}{\partial \mu_s \partial \theta} = \frac{\nu}{\Lambda}\mu_s^{-1}\left(1-\frac{k}{\delta}\right)\left\{\frac{\beta-\nu}{\nu\Gamma}\left[1+\varphi(\theta)\right]^{-2}\frac{\partial \varphi(\theta)}{\partial \theta}h_s(\theta) + \left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\cdot\frac{\varphi(\theta)}{1+\varphi(\theta)}\right]\frac{\partial h_s(\theta)}{\partial \theta}\right\} > 0$$

Hence, SBTC disproportionately increases employment of skilled and unskilled workers in more productive firms, relative to less productive firms.

Prediction 4: *Increased Sorting and Segregation by Skill* – SBTC strengthens the cross-sectional association between productivity and skilled employment shares, provided that firms employ relatively more unskilled than skilled workers at baseline (the empirically relevant case).

Proof: Taking the first-order derivative of (C.13) we get:

$$\frac{\partial}{\partial \mu_s}\left[\frac{h_s(\theta)}{h(\theta)}\right] = b_s\left(1-\frac{k}{\delta}\right)\frac{\nu}{\Lambda}\mu_s^{-1} \cdot \frac{1}{b_s + b_r\varphi(\theta)^{1-\frac{k}{\delta}}} \cdot \frac{h_s(\theta)}{h(\theta)}$$

The second-order derivative yields:

$$\frac{\partial^2}{\partial \mu_s \partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{1}{\left[b_s + b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]^3}\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-\frac{k}{\delta}}\frac{\partial \varphi(\theta)}{\partial \theta}\left[b_s - b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]$$

Given that the ratio of skilled to unskilled workers is $h_s(\theta)/h_r(\theta) = \frac{b_r}{b_s}\varphi(\theta)^{1-\frac{k}{\delta}}$, the term $\left[b_s - b_r\varphi(\theta)^{1-\frac{k}{\delta}}\right]$ is positive if $h_s(\theta)/h_r(\theta) < 1$ and negative if $h_s(\theta)/h_r(\theta) > 1$. Therefore,

$$\frac{\partial^2}{\partial \mu_s \partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] \quad > \quad 0 \quad \text{if} \quad \frac{h_s(\theta)}{h_r(\theta)} < 1 \tag{C.36}$$

Prediction 5: *Differential Wage Growth* – SBTC strengthens the cross-sectional association between productivity and wages conditional on worker skill.

Proof: Taking the first- and second-order derivatives of (C.16) we obtain:

$$\begin{aligned}
\frac{\partial w_r(\theta)}{\partial \mu_s} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\nu}{\delta\Lambda}\mu_s^{-1} w_r(\theta)\frac{\varphi(\theta)}{1+\varphi(\theta)} \\
\frac{\partial^2 w_r(\theta)}{\partial \mu_s \partial \theta} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\nu}{\delta\Lambda}\mu_s^{-1}\frac{1}{1+\varphi(\theta)}\left[\varphi(\theta)\frac{\partial w_r(\theta)}{\partial \theta} + \frac{w_r(\theta)}{1+\varphi(\theta)} \cdot \frac{\partial \varphi(\theta)}{\partial \theta}\right] \quad > \quad 0
\end{aligned}$$

Hence, SBTC disproportionately increases the wages of unskilled workers in more productive firms, relative to less productive firms.

Similarly, taking the derivatives of (C.18):

$$\begin{aligned}
\frac{\partial w_s(\theta)}{\partial \mu_s} &= \frac{k\nu}{\delta\Lambda}\mu_s^{-1} w_s(\theta)\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right] \\
\frac{\partial^2 w_s(\theta)}{\partial \mu_s \partial\theta} &= \frac{k\nu}{\delta\Lambda}\mu_s^{-1}\left\{\frac{\partial w_s(\theta)}{\partial\theta}\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right] + w_s(\theta)\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left[1+\varphi(\theta)\right]^{-2}\frac{\partial\varphi(\theta)}{\partial\theta}\right\} > 0
\end{aligned}$$

Thus, the wages of skilled workers increase disproportionately in more productive firms (relative to less productive firms) as a result of SBTC.

Appendix C.4 Impact of an Increase in the Supply of Skilled Workers

In this section we study the implications of an increase in the supply of skilled workers, modeled as a fall in the search costs of skilled workers. Intuitively, when the supply of skilled workers increases, it becomes easier for firms to fill their vacancies, therefore reducing the cost of searching for workers of this type. For simplicity, we normalize the search cost of unskilled workers to one, i.e. $b_r = 1$, and set $b_s = b$, and study the impacts of a fall in b. It is useful to note that this normalization implies that κ_r, h_{dr} and w_{dr} are independent of b, the search costs for skilled workers. In addition, note that $\varphi(\theta) = \mu_s^{\nu/\Lambda}\left(\frac{\lambda_s}{\lambda_r}\right)^{1/\Lambda} b^{-\gamma\nu/\Lambda}\theta^{\nu/\Lambda}$, which implies $\frac{\partial\varphi(\theta)}{\partial b} < 0$.

In what follows, we proceed in the same order as we did for SBTC, first evaluating how the shock affects firm-level outcomes and then examining the heterogeneity of the effects across the productivity distribution. In doing so we impose some parameter restrictions to ensure that, consistent with the existing literature, wages of skilled workers fall when the supply of skilled workers rises. In particular, we assume that $1 - \frac{k\gamma\nu}{\delta\Lambda}\left(1 + \frac{\beta-\nu}{\nu\Gamma}\right) > 0$.

Prediction 1: *Decreased Skilled Wage Premium Within All Firms* – An increase in the supply of skilled workers decreases the skilled wage premium within all firms.

Proof:

$$\begin{aligned}
\frac{\partial w_r(\theta)}{\partial b} &= -\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\gamma\nu}{\delta\Lambda}b^{-1}\frac{\varphi(\theta)}{1+\varphi(\theta)}w_r(\theta) \quad < \quad 0 \\
\frac{\partial w_s(\theta)}{\partial b} &= w_r(\theta)\varphi(\theta)^{k/\delta}\left\{1-\frac{k\gamma\nu}{\delta\Lambda}\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right]\right\} > 0 \quad \text{if} \quad 1-\frac{k\gamma\nu}{\delta\Lambda}\left(1+\frac{\beta-\nu}{\nu\Gamma}\right) > 0
\end{aligned}$$

When the search cost of skilled workers falls, wages of unskilled workers will increase and (under our parameter restrictions) wages of skilled workers will fall. Intuitively, wages are bargained down to the replacement cost of a worker. This cost is a function of both, (i) the search cost and (ii) a firm's (endogenously chosen) ability threshold, which in turn is positively related to revenue. Thus, a change in the search cost impacts wages directly and, via its effect on the ability threshold, indirectly.

A fall in b makes it cheaper for firms to find skilled workers and hence increases firms' revenues. As a result, firms screen more intensively and choose a higher ability threshold not only for skilled but also for unskilled workers. Thus, even if the search cost of unskilled workers remains unchanged, a higher ability threshold translates into higher unskilled wages. For skilled workers, the wage effects are ambiguous because the direct and indirect wage effects of a fall in b work at cross-purposes. On one hand, the increase in revenue leads firms to choose a higher ability threshold. This effect pushes wages upwards for both skilled and unskilled workers (although to varying extents). On the other hand, skilled workers become cheaper to replace. Given that wages are adjusted down to the replacement cost of workers, this direct effect (which is absent for unskilled types) gives rise to a fall in skilled wages.

Prediction 2: *Selection* – A positive shock to the supply of skilled workers increases θ_d.

Proof: Prediction 2 can be proven by contradiction, as we do for SBTC. Noting that b only appears in $\varphi(\theta)$, it is straightforward to show that a fall in b will increase θ_d, the productivity threshold for production, hence leading to the exit of the least productive firms.

Prediction 3: *Differential Employment Growth* – A positive shock to the supply of skilled workers strengthens the cross-sectional association between employment and productivity.

Proof: Taking the first- and second-order derivatives of (C.10), we obtain:

$$\frac{\partial h_r(\theta)}{\partial b} = -\frac{\gamma\nu}{\Lambda} b^{-1} \left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right) h_r(\theta)\left[1+\varphi(\theta)\right]^{-1}\varphi(\theta) \quad < \quad 0.$$

and recalling that $\frac{\partial\varphi(\theta)}{\partial\theta} > 0$ and noting that $1+\left[\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)-1\right]\frac{\varphi(\theta)}{1+\varphi(\theta)} > 0$:

$$\frac{\partial^2 h_r(\theta)}{\partial b\partial\theta} = \frac{\partial h_r(\theta)}{\partial b}\varphi(\theta)^{-1}\frac{\partial\varphi(\theta)}{\partial\theta}\left\{1+\left[\left(\frac{\beta-\nu}{\nu\Gamma}\right)\left(1-\frac{k}{\delta}\right)-1\right]\frac{\varphi(\theta)}{1+\varphi(\theta)}\right\} \quad < \quad 0$$

Hence, a skill-biased supply shock increases employment of unskilled workers for all firms, but more so for more productive firms. Similarly, taking the derivatives of (C.12) and using the previous results:

$$\frac{\partial h_s(\theta)}{\partial b} = \frac{1}{b}\varphi(\theta)^{1-k/\delta}\left\{\frac{\partial h_r(\theta)}{\partial b} - \frac{1}{b}\left[1+\frac{\gamma\nu}{\Lambda}\left(1-\frac{k}{\delta}\right)\right]\right\} < 0$$

and recalling that $\frac{\partial h_r(\theta)}{\partial \theta} > 0$:

$$\frac{\partial^2 h_s(\theta)}{\partial b \partial \theta} = \left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-1}\frac{\partial \varphi(\theta)}{\partial \theta}\frac{\partial h_s(\theta)}{\partial b} + \varphi(\theta)^{1-k/\delta}\left\{-\frac{1}{b^2}\left[1+\frac{\gamma\nu}{\Lambda}\left(1-\frac{k}{\delta}\right)\right]\frac{\partial h_r(\theta)}{\partial \theta} + \frac{1}{b}\frac{\partial^2 h_r(\theta)}{\partial b \partial \theta}\right\} < 0$$

Hence, a positive shock to the supply of skilled workers also increases employment of skilled workers for all firms, but more so for more productive firms.

Prediction 4: *Increased Sorting and Segregation by Skill* – An increase in the supply of skilled workers strengthens the cross-sectional association between productivity and employment shares of skilled workers.

Proof: Taking the first-order derivative of (C.13), we get:

$$\frac{\partial}{\partial b}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{1}{\left[b+\varphi(\theta)^{1-k/\delta}\right]}\frac{h_s(\theta)}{h(\theta)}\left[\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-1}\frac{\partial \varphi(\theta)}{\partial b}b - 1\right] < 0$$

Hence, a skill-biased supply shock increases the share of skilled workers for all firms. Given our assumption that firm skilled employment is lower than unskilled employment at baseline (i.e. $b - \varphi(\theta)^{1-k/\delta} > 0$) and taking the second-order derivative of the firm employment share of skilled workers, we obtain:

$$\frac{\partial^2}{\partial b \partial \theta}\left[\frac{h_s(\theta)}{h(\theta)}\right] = \frac{\partial}{\partial b}\left[\frac{h_s(\theta)}{h(\theta)}\right] \cdot \frac{1}{\left[b+\varphi(\theta)^{1-k/\delta}\right]}\left(1-\frac{k}{\delta}\right)\varphi(\theta)^{-1}\frac{\partial \varphi(\theta)}{\partial \theta}\left[b-\varphi(\theta)^{1-k/\delta}\right] < 0$$

This result implies that the increase in the share of skilled workers will be larger for more productive firms. Therefore, a positive shock to the supply of skilled workers increases sorting of skilled workers into firms with relatively higher productivity levels.

Prediction 5: *Differential Wage Growth* – An increase in the supply of skilled workers unambiguously leads to an increase in the wages of unskilled workers that is disproportionately larger for more productive firms. Under some reasonable parameter restrictions, wages of skilled workers will fall more for more productive firms.

Proof: Taking the second-order derivative of (C.16) and recalling that $\frac{\partial w_r(\theta)}{\partial \theta} > 0$ and $\frac{\partial \varphi(\theta)}{\partial \theta} > 0$, we obtain:

$$\frac{\partial^2 w_r(\theta)}{\partial b \partial \theta} = -\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k\gamma\nu}{\delta\Lambda}b^{-1}\frac{1}{1+\varphi(\theta)}\left[\varphi(\theta)\frac{\partial w_r(\theta)}{\partial \theta} + \frac{w_r(\theta)}{1+\varphi(\theta)} \cdot \frac{\partial \varphi(\theta)}{\partial \theta}\right] < 0$$

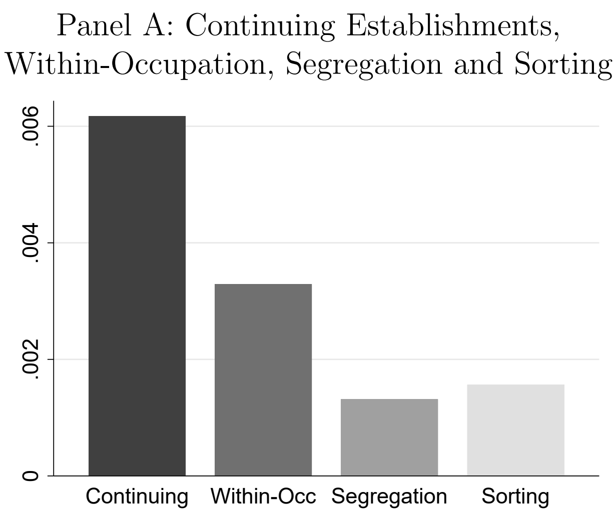
Hence, an increase in the supply of skilled workers unambiguously raises firm wages of unskilled workers (see Prediction 1), and more so for more productive firms.

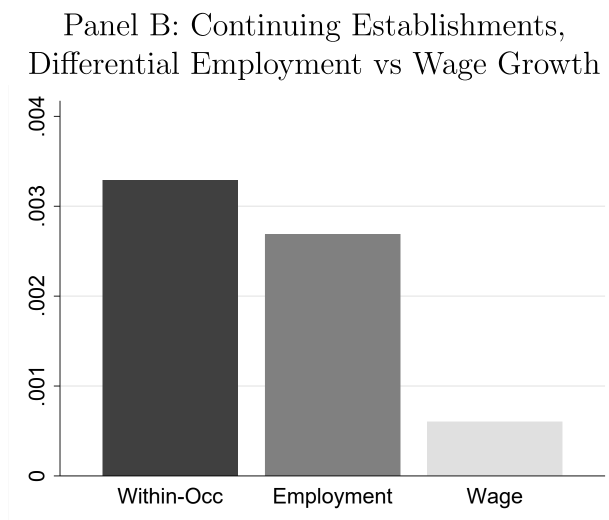
By contrast, whether wage inequality between firms increases for skilled workers depends on parameter restrictions. Taking the second derivative of skilled wages with respect to the search costs yields:

$$\begin{aligned}\frac{\partial^2 w_s(\theta)}{\partial b \partial \theta} &= \left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{k}{\delta}\left[1+\varphi(\theta)\right]^{-1}\frac{\partial\varphi(\theta)}{\partial\theta}w_r(\theta)\varphi(\theta)^{k/\delta}\left\{1-\frac{k\gamma\nu}{\delta\Lambda}\left[1+\left(\frac{\beta-\nu}{\nu\Gamma}\right)\frac{\varphi(\theta)}{1+\varphi(\theta)}\right]-\frac{\gamma\nu}{\Lambda}\left[\frac{k}{\delta}+\frac{1}{1+\varphi(\theta)}\right]\right\} \\ &+ \frac{k}{\delta}\frac{\partial\varphi(\theta)}{\partial\theta}w_r(\theta)\varphi(\theta)^{\frac{k}{\delta}-1}\end{aligned}$$

Under the assumption that $1-\frac{k\gamma\nu}{\delta\Lambda}\left(1+\frac{\beta-\nu}{\nu\Gamma}\right)>0$, wages of skilled workers fall (see Prediction 1). In addition, assuming that $1-\frac{k\gamma\nu}{\delta\Lambda}\left(1+\frac{\beta-\nu}{\nu\Gamma}\right)-\frac{\gamma\nu}{\Lambda}\left(1+\frac{k}{\delta}\right)>0$, wage changes of skilled workers would be disproportionately larger for more productive firms.

Figure A.1: Decomposition of Changes in the Within-Industry Between-Establishment Variance of Log Wages: 3-Digit Occupations





Note: Panel A decomposes changes in the within-industry variance of log wages among continuing establishments into within-occupation, segregation and sorting components; see Equation (9). Panel B decomposes changes in the within-occupation component among continuing establishments into differential employment and wage growth components; see Equation (10). The decompositions are performed over non-overlapping 5-year windows within industries and then averaged across industries and time using 1990 employment shares as weights and giving equal weight to each time period.

Table A.1: Mapping of Occupation Codes to Skill Groups

Skill Group	Occupation Codes (KldB88)	Education Shares (at most high school / university)	Most Common Occupations
Skilled	303, 304, 600-635, 684, 751-763, 811-893	1.91% / 36.97%	nurses (8.68%), managers (7.11%)
Unskilled	71-302, 305-549, 681-683, 685-744, 771-805, 901-937	11.81% / 6.76%	office clerks (16.85%), salespersons (5.87%)
Excluded (Agric/ Forestry/ Unpaid)	11-62, 555, 666, 971-999		

Note: Education shares and shares of most common occupations (shown in brackets) weighted by employment status, with part-time employees counting as 0.5 and full-time employees counting as 1.

Table A.2: Differences between Below and Above Median Industries by Increases in Skilled Employment Shares

	(1)	(2)	(3)	(4)
Panel A: Change in Overall Variance of Establishment (log) Wages				
Difference 2000	0.0090**	0.0089**	0.0092***	0.0090**
	(0.0035)	(0.0035)	(0.0035)	(0.0035)
Difference 2010	0.016***	0.016**	0.016**	0.016**
	(0.0061)	(0.0062)	(0.0062)	(0.0063)
No. of Industries	196	196	196	196
Panel B: Change in Variance of Establishment Wage Premiums (skills)				
Difference 2000	0.0071**	0.0071**	0.0073**	0.0073**
	(0.0034)	(0.0034)	(0.0033)	(0.0033)
Difference 2010	0.013**	0.013**	0.013**	0.013**
	(0.0058)	(0.0059)	(0.0059)	(0.0059)
No. of Industries	196	196	196	196
Panel C: Change in Variance of Establishment Wage Premiums (occup)				
Difference 2000	0.0038*	0.0038*	0.0039*	0.0039*
	(0.0023)	(0.0022)	(0.0022)	(0.0022)
Difference 2010	0.0053	0.0053	0.0052	0.0052
	(0.0037)	(0.0037)	(0.0036)	(0.0037)
No. of Industries	196	196	196	196
Panel D: Change in Variance of Establishment Skill Share				
Difference 2000	0.0021	0.0018	0.0018	0.0017
	(0.0016)	(0.0017)	(0.0016)	(0.0017)
Difference 2010	0.0045**	0.0042*	0.0041*	0.0040*
	(0.0022)	(0.0022)	(0.0022)	(0.0022)
No. of Industries	196	196	196	196
Panel E: Change in Co-variance of Estab. Wage Premiums and Skill Shares				
Difference 2000	0.0016***	0.0016***	0.0016***	0.0015***
	(0.00049)	(0.00049)	(0.00049)	(0.00049)
Difference 2010	0.0030***	0.0029***	0.0029***	0.0028***
	(0.00091)	(0.00093)	(0.00094)	(0.00096)
No. of Industries	196	196	196	196
Controls				
Trade Shock Exposure	no	yes	no	yes
Offshorability	no	no	yes	yes

Note: The table shows the estimates obtained when regressing the industry-level change in the outcome variable of interest on a dummy variable indicating whether the industry is above or below the median in terms of technology adoption (measured as the change in the industry's skilled employment share), plus the control variables indicated at the bottom of the table. 'Difference 2000' is based on a regression that uses the change between 1990 and 2000 as the dependent variable. 'Difference 2010' is based on a regression that uses the change between 1990 and 2010 as the dependent variable. The trade shock exposure controls are the change in per-worker exports and imports with China and Eastern Europe at the industry level between 1990 and 2010. The offshorability control is constructed from the occupational-level offshorability index from Goos et al. (2014), aggregated to the industry level based on the 1990 occupational composition of each industry. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.3: Differences between Below and Above Median Industries by Robot Adoption

	(1)	(2)	(3)	(4)
Panel A: Change in Overall Variance of Establishment (log) Wages				
Difference 2000	0.0038	0.0036	0.0047	0.0045
	(0.0041)	(0.0041)	(0.0041)	(0.0041)
Difference 2010	0.0079	0.0079	0.0081	0.0082
	(0.0069)	(0.0070)	(0.0068)	(0.0068)
No. of Industries	193	193	193	193
Panel B: Change in Variance of Establishment Wage Premiums (skills)				
Difference 2000	0.0013	0.0013	0.0023	0.0022
	(0.0039)	(0.0039)	(0.0038)	(0.0038)
Difference 2010	0.0033	0.0037	0.0041	0.0043
	(0.0066)	(0.0066)	(0.0063)	(0.0063)
No. of Industries	193	193	193	193
Panel C: Change in Variance of Establishment Wage Premiums (occup)				
Difference 2000	0.0026	0.0031	0.0035	0.0037
	(0.0024)	(0.0024)	(0.0026)	(0.0025)
Difference 2010	0.0071**	0.0077**	0.0074*	0.0079**
	(0.0035)	(0.0037)	(0.0039)	(0.0039)
No. of Industries	193	193	193	193
Panel D: Change in Variance of Establishment Skill Share				
Difference 2000	0.0043***	0.0040***	0.0037**	0.0036**
	(0.0013)	(0.0014)	(0.0015)	(0.0016)
Difference 2010	0.0047**	0.0043**	0.0033	0.0033
	(0.0019)	(0.0021)	(0.0024)	(0.0024)
No. of Industries	193	193	193	193
Panel E: Change in Co-variance of Estab. Wage Premiums and Skill Shares				
Difference 2000	0.0013***	0.0012**	0.0011**	0.0011**
	(0.00048)	(0.00050)	(0.00052)	(0.00053)
Difference 2010	0.0029***	0.0026***	0.0026**	0.0025**
	(0.00096)	(0.00100)	(0.0011)	(0.0011)
No. of Industries	193	193	193	193
Controls				
Trade Shock Exposure	no	yes	no	yes
Offshorability	no	no	yes	yes

Note: The table shows the estimates obtained when regressing the industry-level change in the outcome variable of interest on a dummy variable indicating whether the industry is above or below the median in terms of technology adoption (measured in terms of robot adoption in the industry between 1993 and 2010), plus the control variables indicated at the bottom of the table. 'Difference 2000' is based on a regression that uses the change between 1990 and 2000 as the dependent variable. 'Difference 2010' is based on a regression that uses the change between 1990 and 2010 as the dependent variable. The trade shock exposure controls are the change in per-worker exports and imports with China and Eastern Europe at the industry level between 1990 and 2010. The offshorability control is constructed from the occupational-level offshorability index from Goos et al. (2014), aggregated to the industry level based on the 1990 occupational composition of each industry. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.